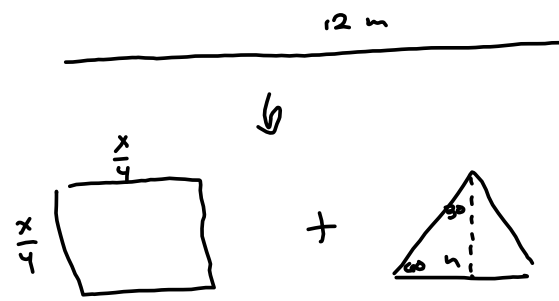


area $= \frac{1}{2} b h$

Cut a wire into 2 pieces. Make one piece a square. The other an equilateral triangle. Find the 2 lengths that maximize the area.

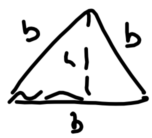
Let x = length of wire used for the square (in m)

y = " " " " " " triangle (in m)

Let A = area of the two figures as a function of x.

Notice $y = 12 - x$

Wheel re-invention for triangle



$\frac{1}{2} b h = \frac{1}{2}\left(\frac{1}{3}y\right)\left(\frac{\sqrt{3}}{6}y\right)$

$\frac{\frac{1}{2}b}{b} = \cos 60^\circ$ duh!

$\frac{h}{b} = \sin(60^\circ) = \frac{\sqrt{3}}{2}$

$h = \frac{\sqrt{3}}{2} b = \frac{\sqrt{3}}{2}\left(\frac{1}{3}y\right)$

$\Longrightarrow \frac{1}{2} b h = \frac{1}{2}\left(\frac{1}{3}y\right)\left(\frac{\sqrt{3}}{2}\left(\frac{1}{3}y\right)\right) = \frac{\sqrt{3}}{36} y^2$ = Area of $\triangle$

Area of square $= \left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$ = area of $\square$

$\Longrightarrow A = \frac{x^2}{16} + \frac{\sqrt{3}}{36} y^2 = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(12-x)^2 = A(x)$

Domain of the problem: $0 \le x \le 12$. Check endpoints?

$$A'(x) = \frac{x}{8} + 2(12-x)'(-1)\frac{\sqrt{3}}{36} = \frac{x}{8} - \frac{\sqrt{3}}{18}(12-x)$$

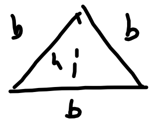
$$= \frac{x}{8} - (24 + 2x)\frac{\sqrt{3}}{36} = \frac{x}{8} - \frac{24\sqrt{3}}{36} - \frac{\sqrt{3}}{18}x \overset{\text{SET}}{=} 0$$

$$\Rightarrow \frac{x}{8} - \frac{\sqrt{3}}{18}x - \frac{2\sqrt{3}}{3} = 0 \Rightarrow$$

$$\Rightarrow \left(\frac{1}{8} - \frac{\sqrt{3}}{18}\right)x = \frac{2\sqrt{3}}{3}$$

$$\Rightarrow \frac{9 - 4\sqrt{3}}{72}x = \frac{2\sqrt{3}}{3}$$

$$\Rightarrow x = \frac{2\sqrt{3}}{3}\left(\frac{72}{9-4\sqrt{3}}\right) = \frac{48\sqrt{3}}{9-4\sqrt{3}}$$



$$\frac{1}{2}bh = \frac{1}{2}\left(\frac{1}{3}y\right)\left(\frac{\sqrt{3}}{2}\left(\frac{1}{3}y\right)\right)$$

$$= \frac{\sqrt{3}}{36}y^2 = \frac{\sqrt{3}}{36}(12-x)^2$$

$$\frac{h}{b} = \sin 60^\circ$$

$$= \frac{\sqrt{3}}{36}(x-12)^2$$

$$h = b\sin 60^\circ = \frac{\sqrt{3}}{2}b$$

$$\frac{\sqrt{3}}{36}y^2 = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(12-x)^2 = A(x)$$

$$\frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$\frac{1}{16}x^2 + \frac{\sqrt{3}}{36}(x^2 - 24x + 144)$$

$$= \frac{1}{16}x^2 + \frac{\sqrt{3}}{36}x^2 - \frac{24\sqrt{3}}{36}x + \frac{144\sqrt{3}}{36} = \left(\frac{1}{16} + \frac{\sqrt{3}}{36}\right)x^2 - \frac{2\sqrt{3}}{3}x + 4\sqrt{3}$$

$$\approx 0.110612522432\,x^2 - 1.15470053838\,x + 6.92820323028$$

(5.21957, 3.91468) $A(5.21957) \approx 3.91468$ MIN.

Max:

$A(0) \approx 6.92820323028$

$\boxed{A(12) = 9 \quad \text{Max}}$

$$\left(\frac{1}{16} + \frac{\sqrt{3}}{36}\right)x^2 - \frac{2\sqrt{3}}{3}x + 4\sqrt{3} = A(x)$$

Vertex: $\left(-\frac{b}{2a}, A\left(-\frac{b}{2a}\right)\right)$

$$-\frac{1}{2}\left(\frac{-\frac{2\sqrt{3}}{3}}{\left(\frac{1}{16}+\frac{\sqrt{3}}{36}\right)}\right) = \frac{2\sqrt{3}}{6}\left(\frac{1}{\left(\frac{1}{16}+\frac{\sqrt{3}}{36}\right)}\right) = \frac{\sqrt{3}}{3}\left(\frac{1}{\frac{9+4\sqrt{3}}{36}}\right)$$

$$= \frac{\sqrt{3}}{3}\left(\frac{36}{9+4\sqrt{3}}\right)$$

$$= \frac{12\sqrt{3}}{9+4\sqrt{3}} = x$$

$2 \cdot 2 \cdot 3 \cdot 3 = 36$

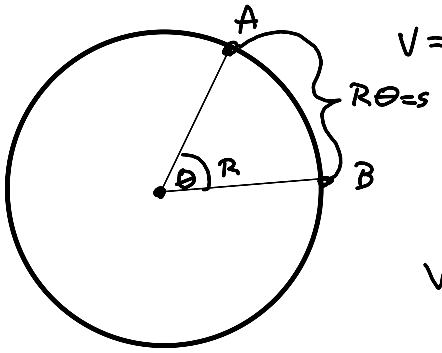
$16 = 2^4$

$LCD = 2^4 \cdot 3 \cdot 3 =$

$$\frac{\cancel{144}^{\cancel{12}^{4}}}{\cancel{36}_{3}}$$

Still off by a factor of 4, but this is the idea.

#10 - Conical Paper drinking cup made from a circular disk of paper with radius *R*.



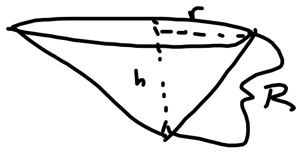
$V = \frac{1}{3}\pi r^2 h$

r = radius of the base

R = radius of paper disk.

s = circumference of the top of cone.

$V = \frac{1}{3}\pi r^2$.



Circumference around the top is $2\pi R - R\theta = (2\pi - \theta)R = 2\pi r$

$$r = \frac{(2\pi-\theta)R}{2\pi} = \left(1 - \frac{\theta}{2\pi}\right)R$$

$h^2 + r^2 = R^2$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(R^2 - h^2)h$$

$$= \frac{1}{3}\pi(R^2 h - h^3)$$

$$\Rightarrow \frac{dV}{dh} = \frac{1}{3}\pi(R^2 - 3h^2) \overset{\text{SET}}{=} 0$$

$$\Rightarrow R^2 = 3h^2 \Rightarrow$$

$$h = \frac{1}{\sqrt{3}}R$$

$$\Rightarrow r^2 = R^2 - h^2 = R^2 - \frac{1}{3}R^2 = \frac{2}{3}R^2$$

$$\Rightarrow r = \sqrt{\frac{2}{3}}\,R$$

Max Volume is $V\left(\frac{1}{\sqrt{3}}R\right) = \frac{1}{3}\pi\left(R^2\left(\frac{1}{\sqrt{3}}R\right) - \left(\frac{1}{\sqrt{3}}R\right)^3\right)$

$$= \frac{1}{3}\pi\left(\frac{1}{\sqrt{3}}R^3 - \frac{1}{3\sqrt{3}}R^3\right) = \frac{\pi}{3}\left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{3}}\right)R^3$$

$$= \frac{\pi}{3}\,\frac{2}{3\sqrt{3}}R^3 = \frac{2\pi\sqrt{3}}{9\sqrt{3}\sqrt{3}}R^3 = \boxed{\frac{2\pi\sqrt{3}}{27}R^3 = V}$$