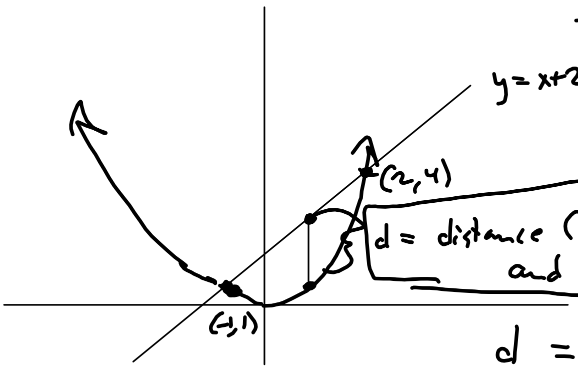
What is the maximum vertical distance between the line $y = x + 2$ and the parabola $y = x^2$ for $-1 \le x \le 2$?

$x = -2!$

$(-2)^2 = 4$

$-2+2=0$



Lexicon (Introduce Notation)

$\boxed{d = \text{distance (vertical) between } y = x+2 \text{ and } y = x^2}$

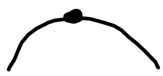
$d = (x+2) - x^2$

$\Rightarrow d' = 1 - 2x \overset{\text{SET}}{=} 0$

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

$\left(\frac{1}{2}, d\left(\frac{1}{2}\right)\right)$



$d\left(\frac{1}{2}\right) = \frac{1}{2} + 2 - \left(\frac{1}{2}\right)^2$

$= \frac{2}{4} + \frac{8}{4} - \frac{1}{4} = \frac{9}{4} = 2{,}25$

Optimization Problems.

1. **Understand the Problem** The first step is to read the problem carefully until it is clearly understood. Ask yourself: What is the unknown? What are the given quantities? What are the given conditions?
2. **Draw a Diagram** In most problems it is useful to draw a diagram and identify the given and required quantities on the diagram.
3. **Introduce Notation** Assign a symbol to the quantity that is to be maximized or minimized (let's call it Q for now). Also select symbols $(a, b, c, \ldots, x, y)$ for other unknown quantities and label the diagram with these symbols. It may help to use initials as suggestive symbols—for example, A for area, h for height, t for time.
4. Express Q in terms of some of the other symbols from Step 3. Auxiliary Equation
5. If Q has been expressed as a function of more than one variable in Step 4, use the given information to find relationships (in the form of equations) among these variables. Then use these equations to eliminate all but one of the variables in the expression for Q. Thus Q will be expressed as a function of *one* variable x, say, $Q = f(x)$. Write the domain of this function in the given context.
6. Use the methods of Sections 3.1 and 3.3 to find the *absolute* maximum or minimum value of f. In particular, if the domain of f is a closed interval, then the Closed Interval Method in Section 3.1 can be used.

Consider the following problem: A farmer with 850 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

(a) Draw several diagrams illustrating the situation, some with shallow, wide pens and some with deep, narrow pens. Find the total areas of these configurations. Does it appear that there is a maximum area? If so, estimate it.

(b) Draw a diagram illustrating the general situation. Let x denote the length of each of two sides and three dividers. Let y denote the length of the other two sides.

(c) Write an expression for the total area A in terms of both x and y.

A = ✗ xy

(d) Use the given information to write an equation that relates the variables.

✗ $5x + 2y = 850$

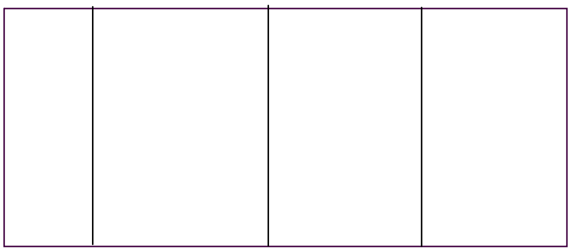
(e) Use part (d) to write the total area as a function of one variable.

$A(x)$ = ✗ $425x - \frac{5x^2}{2}$

a

850 ft of fence. Divide into 4 pens with sides parallel to the ends. whats largest total area of the pens?

x



y

x = width of pen in feet , y = length of pen in ft.

Fence used = 850 ft = $5x + 2y$ — Auxiliary Equation

Area = xy = Q = area in ft^2

$5x+2y = 850$

$2y = 850 - 5x$

$y = 425 - \frac{5}{2}x$

Let $Q = Q(x)$ = Area enclosed by the fencing.

want ?

$\Rightarrow Q = Q(x) = xy = x\left(425 - \frac{5}{2}x\right) = 425x - \frac{5}{2}x^2$

$\Rightarrow Q'(x) = 425 - 5x \overset{\text{SET}}{=} 0 \Rightarrow$

$5x = 425 \longrightarrow$

$x = \frac{425}{5} = 85\text{ft} \longrightarrow y = 425 - \frac{5}{2}x = 425 - \frac{5}{2}(85) = \frac{850-425}{2} = \frac{425}{2} = \boxed{y = 212.5 \text{ ft}}$

$\boxed{x = 85 \text{ ft}}$

$Q(85) = 425(85) - \frac{5}{2}(85)^2$

425 × 85 = 2125 + 34000 = 36125

85 × 85 = 425 + 6800 = 7225; 7225 × 5 = 36125

$\frac{36125}{2} = \boxed{18062.5 \text{ ft}^2}$ $\boxed{= Q(85) \text{ is a max.}}$

18062.5 + 36125.0 = 54187.5

$-\frac{5}{2}x^2 + 425x$

$$425x - \frac{5}{2}x^2 = -\frac{5}{2}x^2 + 425x$$

$$\frac{2}{5}(425) = 2(85) = 170$$

$$= -\frac{5}{2}\left(x^2 - 170x + (85)^2\right) + \frac{36125}{2}$$

$$= -\frac{5}{2}(x - 85)^2 + \frac{36125}{2}$$

$$\left(\frac{5}{2}\right)(85)^2$$

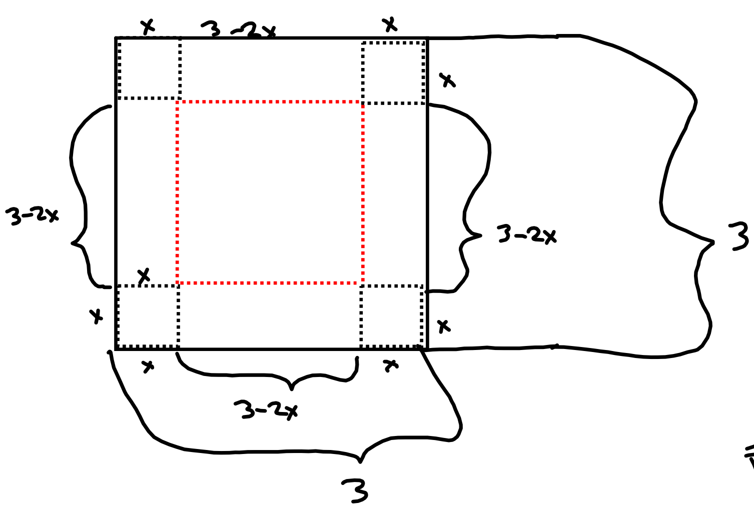
h k

$$(h, k) = \left(85, \frac{36125}{2}\right)$$

$$\left(\frac{5}{2}\right)(7225) = \frac{36125}{2}$$

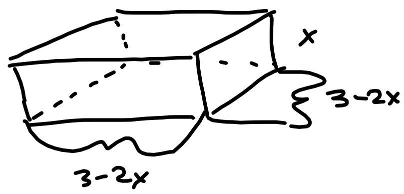
$$\begin{array}{r} 85 \\ \times\ 85 \\ \hline 425 \\ 6800 \\ \hline 7225 \end{array}$$

4. Consider the following problem: A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.



We cut the corners out of a square piece of cardboard that's 3ft x 3ft

This is a college-algebra question.



Let $\boxed{V = \text{Volume of the box in } ft^3, \text{ as a function of } x = \text{the side-lengths of the squares we cut out of the corners of the cardboard in ft.}}$

Then $V = V(x) = (3-2x)(3-2x)x = (9-12x+4x^2)\,x$

(I'd just take the derivative, as is.)

$= \boxed{4x^3 - 12x^2 + 9x = V(x)} \Rightarrow$

$V'(x) = 12x^2 - 24x + 9 \overset{SET}{=} 0 \longrightarrow$

$\longrightarrow 4x^2 - 8x + 3 = 0$

$\longrightarrow a = 4, b = -8, c = 3 \longrightarrow$

$b^2 - 4ac = 8^2 - 4(4)(3) = 64 - 48 = 16$

$\& \sqrt{16} = 4 \longrightarrow$

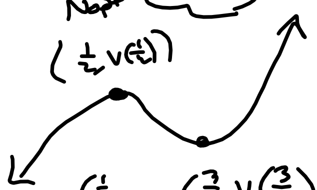
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm 4}{2(4)} = \frac{8 \pm 4}{8} = \frac{2 \pm 1}{2} \to \frac{3}{2}, \frac{1}{2}$

$\longrightarrow x = \frac{3}{2}$ or $\boxed{x = \frac{1}{2}}$ Yes

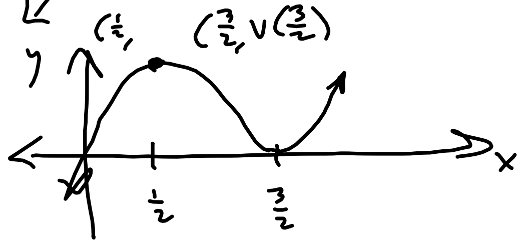
Nope

$0 \leq x \leq \frac{3}{2}$

$(\frac{1}{2}, V(\frac{1}{2}))$



x^3 shape



$V(\frac{1}{2}) =$

$= \boxed{4x^3 - 12x^2 + 9x = V(x)} \longrightarrow$

$$\begin{array}{r|rrrr} \frac{1}{2} & 4 & -12 & 9 & 0 \\ & & 2 & -5 & 2 \\ \hline & 4 & -10 & 4 & 2 \end{array} = V(\tfrac{1}{2})$$

$V(\frac{1}{2}) = 2\ ft^3$ maximizes the volume.