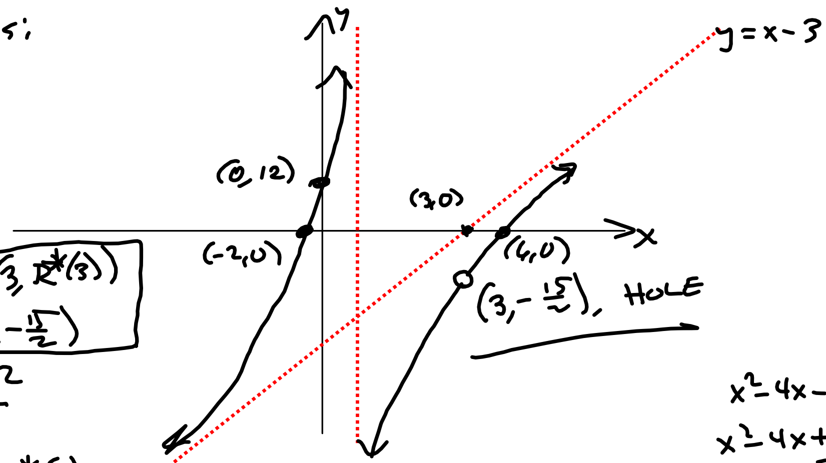
Key Points:

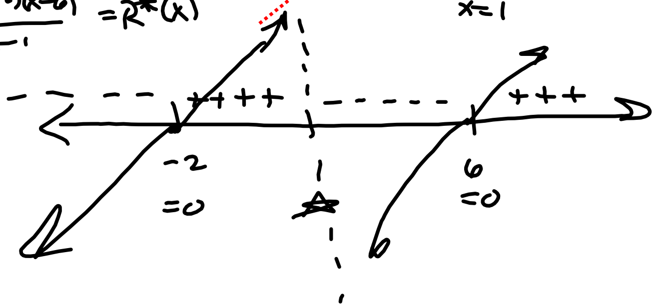
R(x) has a

$\boxed{\text{Hole @ } \left(3, R^*(3)\right) = \left(3, -\tfrac{15}{2}\right)}$



$$\frac{x^2-4x-12}{x-1}$$

$$= \frac{(x+2)(x-6)}{x-1} = R^*(x)$$



$x^2-4x-12=0$

$x^2-4x+2^2=12+4$

$(x-2)^2=16$

$x-2=\pm 4$

$x=6,-2$ ✓

$f(x) = \cos(x) + x$ No paper-and-pencil technique for solving $f(x) = 0$

usually requires technology or a recursive solution, like -0.7390851332

guess-and-check OR NEWTON'S METHOD (End of C3)

$g(x) = \sin(x) + x$ is nicer, as $f(0) = 0$ by inspection!

We graph $g(x)$, b/c I'm weak.

Domain $= \mathbb{R}$. g cnt^s^ & difb^l^ $\forall x \in \mathbb{R}$.

No asymptotes.

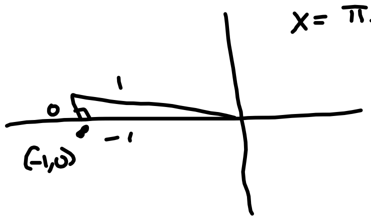
x & y-int are $(0,0)$. Not sure if $\exists$ other x-ints. Calculus will help.

x-int : $(0,0)$ (May be others, but we have this one.)

$g'(x) = \cos(x) + 1 \overset{SET}{=} 0 \longrightarrow$

$\cos(x) = -1$

$x = \pi + 2n\pi \quad \forall n \in \mathbb{Z}$

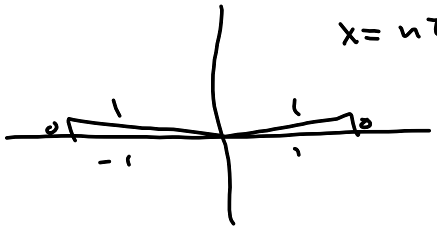


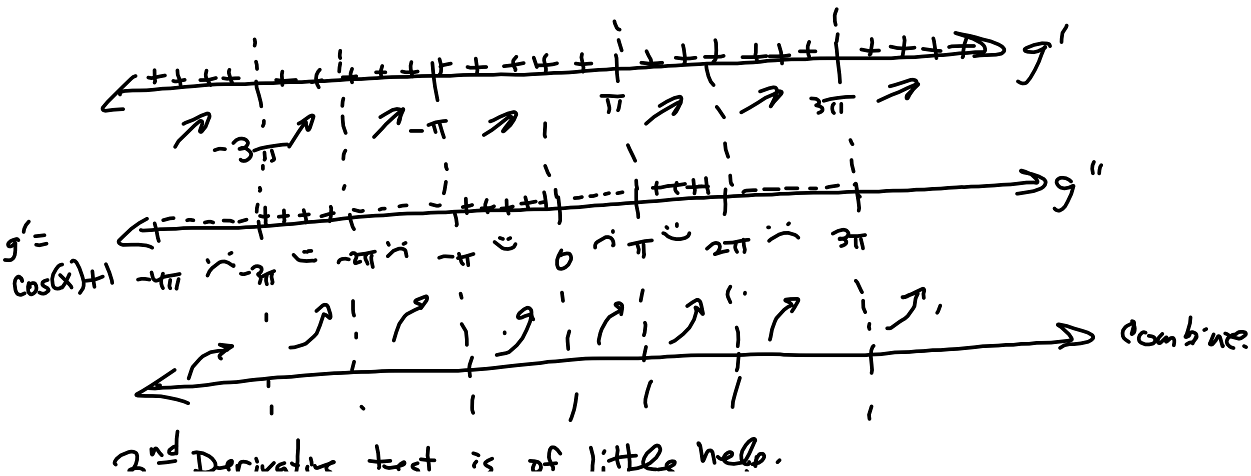
$g''(x) = -\sin(x) \overset{SET}{=} 0 \longrightarrow$

$x = n\pi \quad \forall n \in \mathbb{Z}$

Half of g'''s zeros are also zeros of g'!

That means where $g'(x) = 0$, we'll have terraces, rather than





2nd Derivative test is of little help.

The following would be done on a separate sheet of paper. Screen space is limited.

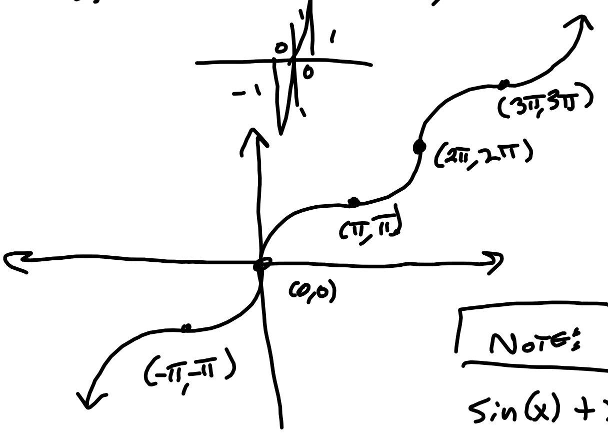
g': Test $x = -4\pi, -2\pi, 0, 2\pi, 4\pi$ Looks like No Max/min values!

$\cos(-4\pi) = \cos(-4\pi) + 1 = 2$, $\cos(-2\pi) + 1 = 2$, They're ALL $= 2$!

$g'' = -\sin(x)$

g'': Test $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$-\sin(\frac{\pi}{2}) = -1$; $\sin(\frac{3\pi}{2}) = -(-1)$; $\sin(\frac{5\pi}{2}) = 1, \ldots$ I think I trust the pattern



$g(x) = \sin(x) + x$

$g(\pi) = 0 + \pi$

$g(2\pi) = 2\pi$

$\boxed{\text{NOTE:}}$

$\sin(x) + x$ is ODD FUNCTION!

$-x + \sin(-x) = -\sin(x) - x = f(-x) = -(\sin(x) + x)$

$= -f(x)$

Formalities;

1st Derivative Test: If c is a critical point for $f(x)$, and $f'(x) < 0$ for $x < c$ (on some neighborhood of c) and $f'(x) > 0$ for $x > c$, then $(c, f(c))$ is a local minimum.

$f(c)$ .. ..

$\searrow | \nearrow$

Likewise, $f' > 0$ for $x < c$ and $f' < 0$ for $x > c$ $\longrightarrow$ $(c, f(c))$ is a local maximum

$\nearrow | \searrow$

c

If $f'(c) = 0$ and $f''(c) > 0$, then $(c, f(c))$ is a local minimum

.. .. $<$ .. .. maximum

.. .. $f''(c) = 0$, then we don't know.

Min, $f'' > 0$ Max, $f'' < 0$

If it's easier to do *f* '' than to do a sign pattern for *f*, then 2nd-derivative test is fine.

Newton's Method.

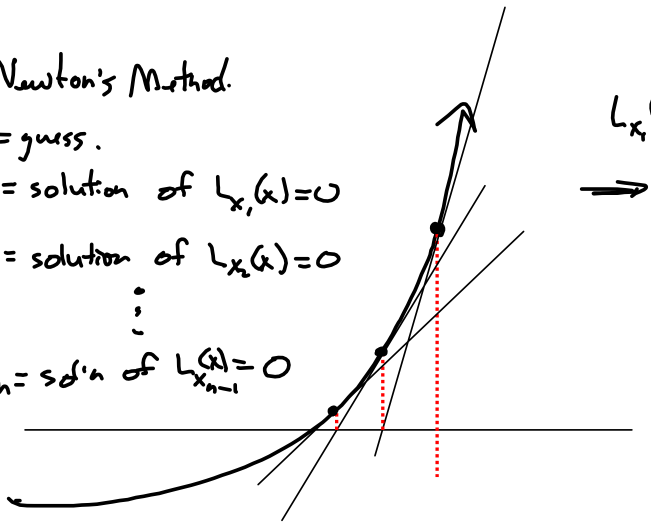
x_1 = guess.

x_2 = solution of $L_{x_1}(x)=0$

x_3 = solution of $L_{x_2}(x)=0$

$\vdots$

x_n = sol'n of $L_{x_{n-1}}(x)=0$



$$L_{x_1}(x) = f'(x_1)(x-x_1)+f(x_1) \overset{SET}{=} 0$$

$$\Longrightarrow f'(x_1)x - f'(x_1)x_1 = -f(x_1)$$

$$f'(x_1)x = f'(x_1)x_1 - f(x_1)$$

$$x = \frac{f'(x_1)x_1}{f'(x_1)} - \frac{f(x_1)}{f'(x_1)}$$

$$x = x_1 - \frac{f(x_1)}{f'(x_1)} = x_2$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\vdots$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Newton's Method

§3.8