Midterm Curve

Off the top:

-5pts #7b 2 variables indicated. Possible bonus, but no penalty if wrong.

-5pts #8? $\sqrt{\frac{2}{\pi}}\,x$ confused people. Also, fairly high difficulty level.

-10pts #9 Gave values in inches. Asked for answers in meters. Poorly posed. You can earn points but not lose any.

NOTE: I may have graded #9 incorrectly. I will go back over everyone's #9 to see if I owe any points.

100

90

76

75

64

63

63 C

55

54

44 D

A - 76 - 105

B - 64 - 75

C 55 - 63

D 44 - 54

A - 76 - 105 (76, 90), (105, 100)

$y = \frac{100-90}{105-76}(x-105)+100 = \boxed{\frac{10}{29}(x-76)+90 = y}$

B - 64 - 75 (64, 80), (75, 89)

$y = \frac{89-80}{75-64}(x-64)+80 = \boxed{\frac{9}{11}(x-64)+80 = y}$

C - 55 - 63 (55, 70), (63, 79)

$y = \frac{79-70}{63-55}(x-55)+70 = \boxed{\frac{9}{8}(x-55)+70 = y}$

Let x = raw score
and y = curved score.

D - 44-54 $(44,60), (54,69)$

$$y = \frac{69-60}{54-44}(x-44)+60 = \boxed{\frac{9}{10}(x-44)+60 = y}$$

F - (0,43) - $(0,0), (43,59)$

$$\boxed{y = \frac{59}{43}x}$$

=if(x<44, 59/43*x, if(x<55, 9/10*(x-44)+60,
if(x<64 9/8*(x-55)+70, if(x<76, 9/11*(x-64)+80,
10/9*(x-76)+90))))

Toughest Questions on the Midterm.

$\boxed{\sin(x^2+y^2)} = \sqrt{\frac{2}{\pi}}\, x + 2y$

$f(g(x))$

$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg} \cdot \frac{dg}{dx} = f'(g)\,g'(x)$

$f(g) = \sin(g) \Rightarrow f'(g) = \cos(g) = \frac{df}{dg}$

$g(x) = x^2 + y^2 = x^2 + y(x)^2$

$y(x)^2 = j(k(x))$ where

$j(k) = k^2 \Rightarrow \frac{dj}{dk} = 2k$

$k(x) = y(x) \Rightarrow k'(x) = y'(x)$

$\Rightarrow g'(x) = 2x + 2k(x)\,k'(x)$

$= 2x + 2y(x) \cdot y'(x)$

$= 2x + 2yy'$

NO

$f(x) = x^2 - 5x + 2 = 2x - 5$

$f(x) = x^2 - 5x + 2 \Rightarrow$

$f'(x) = 2x - 5$

$$\sin(x^2+y^2) = \sqrt{\tfrac{2}{\pi}}\, x + 2y \longrightarrow$$

$$(\cos(x^2+y^2))(2x+2yy') = \sqrt{\tfrac{2}{\pi}} + 2y' \longrightarrow$$

$$2x\cos(x^2+y^2) + 2yy'\cos(x^2+y^2) = \sqrt{\tfrac{2}{\pi}} + 2y' \longrightarrow$$

$$-2x\cos(x^2+y^2) - 2y' \qquad = \qquad -2y' - 2x\cos(x^2+y^2)$$

$$2yy'\cos(x^2+y^2) - 2y' = \sqrt{\tfrac{2}{\pi}} - 2x\cos(x^2+y^2) \longrightarrow$$

$$y'(2y\cos(x^2+y^2) - 2) = \sqrt{\tfrac{2}{\pi}} - 2x\cos(x^2+y^2) \longrightarrow$$

$$\boxed{y' = \frac{\sqrt{\tfrac{2}{\pi}} - 2x\cos(x^2+y^2)}{2y\cos(x^2+y^2) - 2}}.$$

$$y'\Big|_{\left(\sqrt{\frac{\pi}{2}},\, 0\right)} = \frac{\sqrt{\tfrac{2}{\pi}} - 2\sqrt{\tfrac{\pi}{2}}\cos\left(\sqrt{\tfrac{\pi}{2}}^{\,2} + 0^2\right)}{2(0) - 2} = \frac{\sqrt{\tfrac{2}{\pi}}}{-2} = -\frac{1}{2}\sqrt{\tfrac{2}{\pi}}$$

$$= m \longrightarrow$$

$$L(x) = m(x - x_1) + y_1 \quad = \boxed{-\tfrac{1}{2}\sqrt{\tfrac{2}{\pi}}\left(x - \sqrt{\tfrac{2}{\pi}}\right) = L(x)}$$

$$(x_1, y_1) = \left(\sqrt{\tfrac{2}{\pi}}, 0\right)$$

$(x^a)^b = x^{ab}$

$(x^2)^3 = x^{10}$

$\sqrt[5]{x^2} = x^{\frac{2}{5}}$

$\sqrt[8]{x^5} = x^{\frac{5}{8}}$

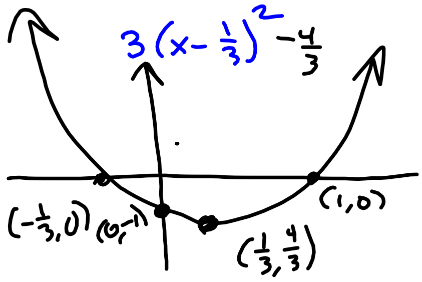
$x^{-2} = \frac{1}{x^2}$

$\frac{11}{x^{-2/3}} = 11x^{2/3}$

$\frac{1}{x^{-2}} = x^2$

$f(x) = 3x^2 - 2x - 1 = 3\left(x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2\right) - 1 - 3\left(\frac{1}{9}\right)$

$3\left(x - \frac{1}{3}\right)^2 - \frac{4}{3}$



$-1 - \frac{1}{3}$

$= -\frac{3}{3} - \frac{1}{3} = -\frac{4}{3}$

$3\left(x - \frac{1}{3}\right)^2 - \frac{4}{3}$ SET 0

$3\left(x - \frac{1}{3}\right)^2 = \frac{4}{3}$

$\left(x - \frac{1}{3}\right)^2 = \frac{4}{9}$

$x - \frac{1}{3} = \pm\sqrt{\frac{4}{9}}$

$x = \frac{1}{3} \pm \frac{2}{3} \rightarrow 1, -\frac{1}{3}$

$\lim_{x\to\infty} f(x) = L$ means $\forall\, \varepsilon > 0, \exists\, N > 0 \ni$

$|f(x) - L| < \varepsilon \quad \forall\, x > N.$ "Eventually, $|f(x)-L|$ can be made smaller than any given positive real #."

3.4 #23 $\lim_{x\to-\infty} \frac{1-3x}{\sqrt{x^2+1}} = -3$

Let $\varepsilon = 0.1$. we find the smallest integer N such that

$\left| \frac{1-3x}{\sqrt{x^2+1}} - (-3) \right| < 0.1$ iff

$\left| \frac{1-3x}{\sqrt{x^2+1}} + 3 \right| < 0.1$

Graphing Calculator recommended.

I tried doing it by hand during in-class work, but failed.

Deeper Analysis:

$$\frac{1-3x}{\sqrt{x^2+1}} = f(x) = \frac{1-3x}{(x^2+1)^{1/2}} = (1-3x)(x^2+1)^{-\frac{1}{2}}$$

$$f'(x) = -3(x^2+1)^{-\frac{1}{2}} + (1-3x)\left(-\frac{1}{2}(x^2+1)^{-\frac{3}{2}}(2x)\right)$$

$$= \frac{-3}{\sqrt{x^2+1}} - \frac{(1-3x)(x)}{\sqrt{x^2+1}\,(x^2+1)}$$

$$= \frac{-3(x^2+1)+3x^2-x}{(x^2+1)^{3/2}} = \frac{-3x^2-3+3x^2-x}{(x^2+1)^{3/2}} = \frac{-(x+3)}{(x^2+1)^{3/2}}$$

$f''(x)$

This is enough to prove that f doesn't increase back out of the ε-tube.

This is enough to trust

$$\boxed{\text{that } \left|\frac{1-3x}{\sqrt{x^2+1}} - (-3)\right| < .1 \quad \forall x > 12}$$