$\sqrt[8]{x^5} = x^{\frac{5}{8}}$

$\frac{11}{x^{-2/3}} = 11x^{\frac{2}{3}}$ $\qquad (x^b)^c = x^{bc}$

differentiate

$f(x) = \sqrt[8]{x^5} = (x^5)^{\frac{1}{8}} = x^{\frac{5}{8}} \longrightarrow f'(x) = \frac{5}{8}x^{-3/8}$

$\Longrightarrow f'(x) = \frac{1}{8}(x^5)^{-7/8}(5x^4) = (\frac{1}{8})(5)\, x^{-35/8} \cdot x^4$

$= \frac{5}{8}x^{\frac{-35+32}{8}} = \frac{5}{8}x^{-\frac{3}{8}}$ $\qquad \frac{32}{8}$

We've been graphing functions with f', f'' tools, but it's all ~~been~~ intuitive. $\ni$

$\boxed{D}$ A <u>critical point</u> c is a number such that $c \in D(f)$, and either $f'(c) = 0$ or $f'(c)$ does not exist.

We use critical points to find extreme values.

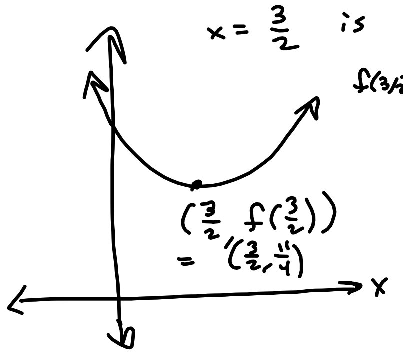
$f(x) \rightsquigarrow f'(x) \overset{\text{SET}}{=} 0$ Also look for zeros in the denominator.

$f(x) = x^2 - 3x + 5 \longrightarrow$

$f'(x) = 2x - 3 \overset{\text{SET}}{=} 0$

$\Longrightarrow 2x = 3$

$x = \frac{3}{2}$ is a critical #.



$f(3/2) = (\frac{3}{2})^2 - 3(\frac{3}{2}) + 5$

$= \frac{9}{4} - \frac{9}{2} \cdot \frac{2}{2} + (5)(\frac{4}{4})$

$= \frac{9 - 18 + 20}{4} = \frac{11}{4}$

$x^2 - 3x + 5 = x^2 - 3x + (\frac{3}{2})^2 - \frac{9}{4} + (5)(\frac{4}{4})$

$= (x - \frac{3}{2})^2 + \frac{11}{4}$

$x^2 - 2x - 15 = (x-5)(x+3) = x^2 - 2x - 15$

$(x+5)(x-3)$

$\lim_{x \to 5^-} \frac{x^2 - 2x - 15}{|x-5|} = \frac{(x+5)(x-3)}{-(x-5)}$

$f(x) = (x-2)^{2/5} \longrightarrow$

$f'(x) = \frac{2}{5}(x-2)^{-\frac{3}{5}} = \frac{2}{5(x-2)^{3/5}} \overset{\text{SET}}{=} 0$ No sol'n

f' $\nexists$ @ $x=2 \longrightarrow x=2$ is a critical #

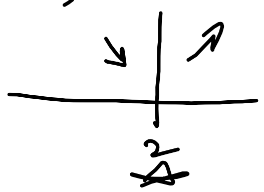
$- - - - - - - \mid + + + + + + + +$

$\cancel{2}$ 2

Put in calculator if you're not confident.

	Test	
$(-\infty, 2)$	0	$\frac{2}{5(0-2)^{3/5}} = \frac{2}{5(-2)^{3/5}} < 0$
$(2, \infty)$	3	$\frac{2}{5(+1)^{3/5}} = +\frac{2}{5} > 0$

1 is better test value



$f'(x) = \frac{2}{5}(x-2)^{-3/5} \longrightarrow$

$f''(x) = \frac{-6}{25}(x-2)^{-\frac{8}{5}}$

$- - - - - - - \mid - - - - - - -$

☹ 2 ☹

	Test	
$(-\infty, 2)$	1	$-\frac{6}{25}(-1)^{-\frac{8}{5}} = -\frac{6}{25}$
$(2, \infty)$	3	$-\frac{6}{25}(1)^{-8/5} = -\frac{6}{25}$

$\frac{-6}{25}(-1)^{-\frac{8}{5}}$

$= \frac{-6}{25((-1)^8)^{1/5}}$

$\boxed{\text{Cusp}}$

Min @ $x=2$

$$\lim_{x\to\infty} \frac{6x^2-5}{7x^2+x-3} = \lim_{x\to\infty} \frac{\cancel{x^2}\left(6-\frac{5}{x^2}\right)}{\cancel{x^2}\left(7+\frac{1}{x}-\frac{3}{x^2}\right)} = \frac{6}{7}$$

$\frac{5}{x^2} \to 0$, $\frac{1}{x} \to 0$, $\frac{3}{x^2} \to 0$

$= y$ is horiztal asymptote

$$= \frac{6}{7}$$

$\lim_{x\to\infty} \frac{x-6}{x^2+2x+13}$ is "proper" $= 0$

Degree of denom. > degree of numer.

$$= \lim_{x\to\infty} \frac{x^2\left(\frac{1}{x}-\frac{6}{x^2}\right)}{x^2\left(1+\frac{2}{x}+\frac{13}{x^2}\right)} = \frac{0}{1} = 0$$

$(x \neq 0)$

$$\lim_{x\to\infty} \frac{x^4}{\sqrt{x^8+7}} = \lim_{x\to\infty} \frac{x^4}{x^4\sqrt{1+\frac{7}{x^8}}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{7}{x^8}}} = \frac{1}{\sqrt{1+0}} = 1$$

$$\sqrt{x^8\left(1+\frac{7}{x^8}\right)} = \sqrt{x^8}\sqrt{1+\frac{7}{x^8}} = |x^4|\sqrt{1+\frac{7}{x^8}} = x^4\sqrt{1+\frac{7}{x^8}}$$

$$\lim_{x\to\infty}\left(\sqrt{64x^2+x} - 8x\right)$$

$$= \lim_{x\to\infty}\left(8x\sqrt{1+\frac{x}{64x^2}} - 8x\right) = \lim_{x\to\infty}\left(8x\left(\sqrt{1+\frac{1}{64x}} - 1\right)\right)$$

$|x| = x$, b/c $x \to \infty$
$\Longrightarrow x > 0$, everywhere, here

$= \infty \cdot 0$? Indeterminate form.

Here's the trick:

$$\left(\sqrt{64x^2+x} - 8x\right)\left(\frac{\sqrt{64x^2+x}+8x}{\sqrt{64x^2+x}+8x}\right)$$

$$= \frac{64x^2+x-64x^2}{\sqrt{64x^2+x}+8x} = \frac{x}{\left(8x\sqrt{1+\frac{1}{64x}}+8x\right)} = \frac{x}{8x\left(\sqrt{1+\frac{1}{64x}}+1\right)}$$

$$= \frac{1}{8\left(\sqrt{1+\frac{1}{64x}}+1\right)} \xrightarrow{x\to\infty} \frac{1}{8(\sqrt{1}+1)} = \frac{1}{8(2)} = \frac{1}{16}$$

$(x \neq 0)$

$\lim_{x \to \infty} \cos(x) \ \nexists$

$\lim_{x \to \infty} \left(\cos\left(\frac{1}{x}\right)\right) = 1$

Find Horizontal Asymptotes (HA!) an Vertical Asymptotes (VA)

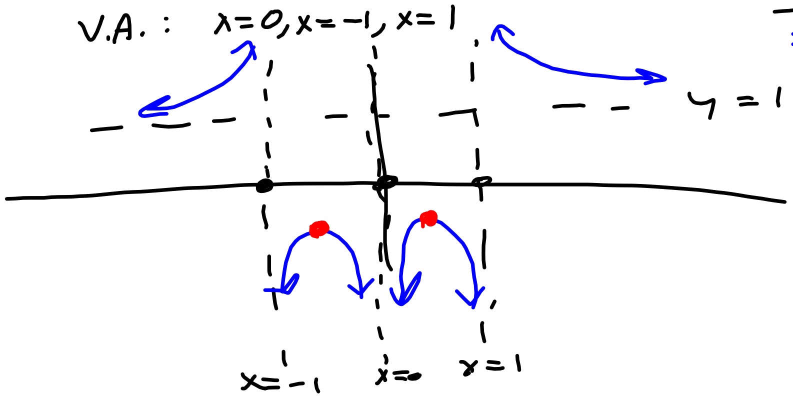
$\frac{x^4+7}{x^4-x^2} \xrightarrow{x \to \infty} 1$ $\boxed{HA: y=1}$

$+++ \ |_{-1} --- |_{0} --- |_{1} \ +$

VA: D: $x^4-x^2 = x^2(x^2-1) = x^2(x+1)(x-1)$

$\frac{x^4+7}{x^2(x-1)(x+1)}$

V.A.: $x=0, x=-1, x=1$



Next Up: Summary of Curve Sketching: Section 3.5.

Technology and Graphing (GIGO): Section 3.6

3.7 - Optimization Problems

3.8 Newton's Method

3.9 - Antiderivatives