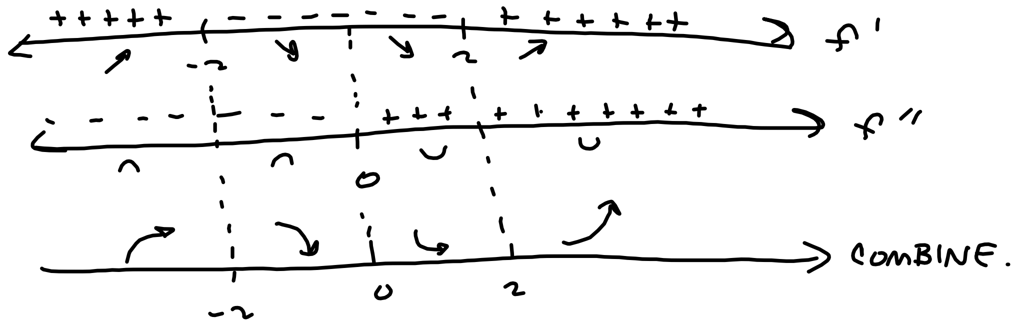
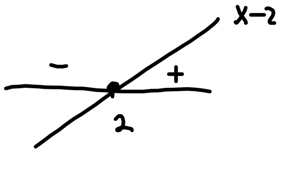
$f(x) = x^3 - 12x + 2$

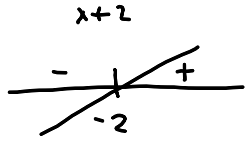
$f'(x) = 3x^2 - 12 = 3(x^2 - 4) = 3(x-2)(x+2) = 0 \rightarrow x \in \{-2, 2\}$

$f''(x) = 6x = 0 \Rightarrow x \in \{0\}$



f':	$x-2$	$x+2$	f'	
$x < -2$	$-$	$-$	$+$	$(-\infty, -2)$
$-2 < x < 2$	$-$	$+$	$-$	$(-2, 2)$
$2 < x$	$+$	$+$	$+$	$(2, \infty)$





Graph One Period!

$f(x) = (\sin(x) - 1)(2\sin(x) + 1)$

$= 2\sin^2 x - \sin(x) - 1$

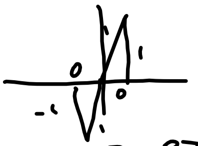
$\Rightarrow f'(x) = 4\sin(x)\cos(x) - \cos(x)$

$= \cos(x)(4\sin(x) - 1) \overset{SET}{=} 0$

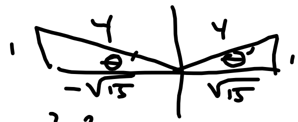
$\Rightarrow \cos(x) = 0$ or $4\sin(x) - 1 = 0$

$4\sin(x) = 1$

$\sin(x) = \frac{1}{4}$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$



$4^2 - 1^2$

$= 15$

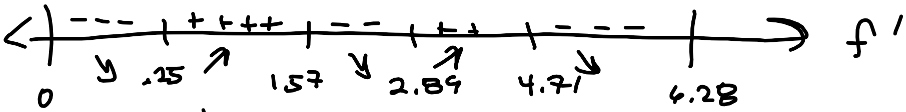
$x = \arcsin\left(\frac{1}{4}\right)$, $\pi - \arcsin\left(\frac{1}{4}\right)$

$\theta' = \arcsin\left(\frac{1}{4}\right)$

θ' = reference angle

zeros of f': $\frac{\pi}{2}, \frac{3\pi}{2}, \arcsin\left(\frac{1}{4}\right), \pi - \arcsin\left(\frac{1}{4}\right)$

$\approx 1.57, 4.71, .25, 2.89,$



	Test	
(0, .25)	.1	-.6
(.25, 1.57)	.5	.8
(1.57, 2.88)	2	-1.1
(2.88, 4.71)	3	.4
(4.71, 6.28)	5	-1.37

2π

$f'(x) = 4\sin(x)\cos(x) - \cos(x) \longrightarrow$

$$f''(x) = 4\cos^2(x) + 4\sin(x)(-\sin(x)) + \sin(x)$$

$$= 4\cos^2(x) - 4\sin^2(x) + \sin(x)$$

$$= 4(1-\sin^2(x)) - 4\sin^2(x) + \sin(x)$$

$$= 4 - 4\sin^2(x) - 4\sin^2(x) + \sin(x)$$

$$= -8\sin^2(x) + \sin(x) + 4 \overset{\text{SET}}{=} 0$$

$$\Rightarrow -8u^2 + u + 4 = 0$$

$$\Rightarrow 8u^2 - u - 4 = 0$$

$a=8, b=-1, c=-4$

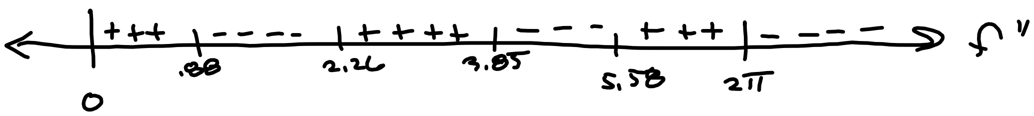
$$\Rightarrow b^2 - 4ac = 1 - 4(8)(-4) \qquad 3 \overline{)129} \; 43$$

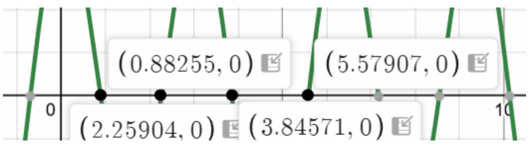
$$= 1 + 128 = 129$$

$$u = \frac{-1 \pm \sqrt{129}}{2(4)} = \frac{-1 \pm \sqrt{129}}{8}$$

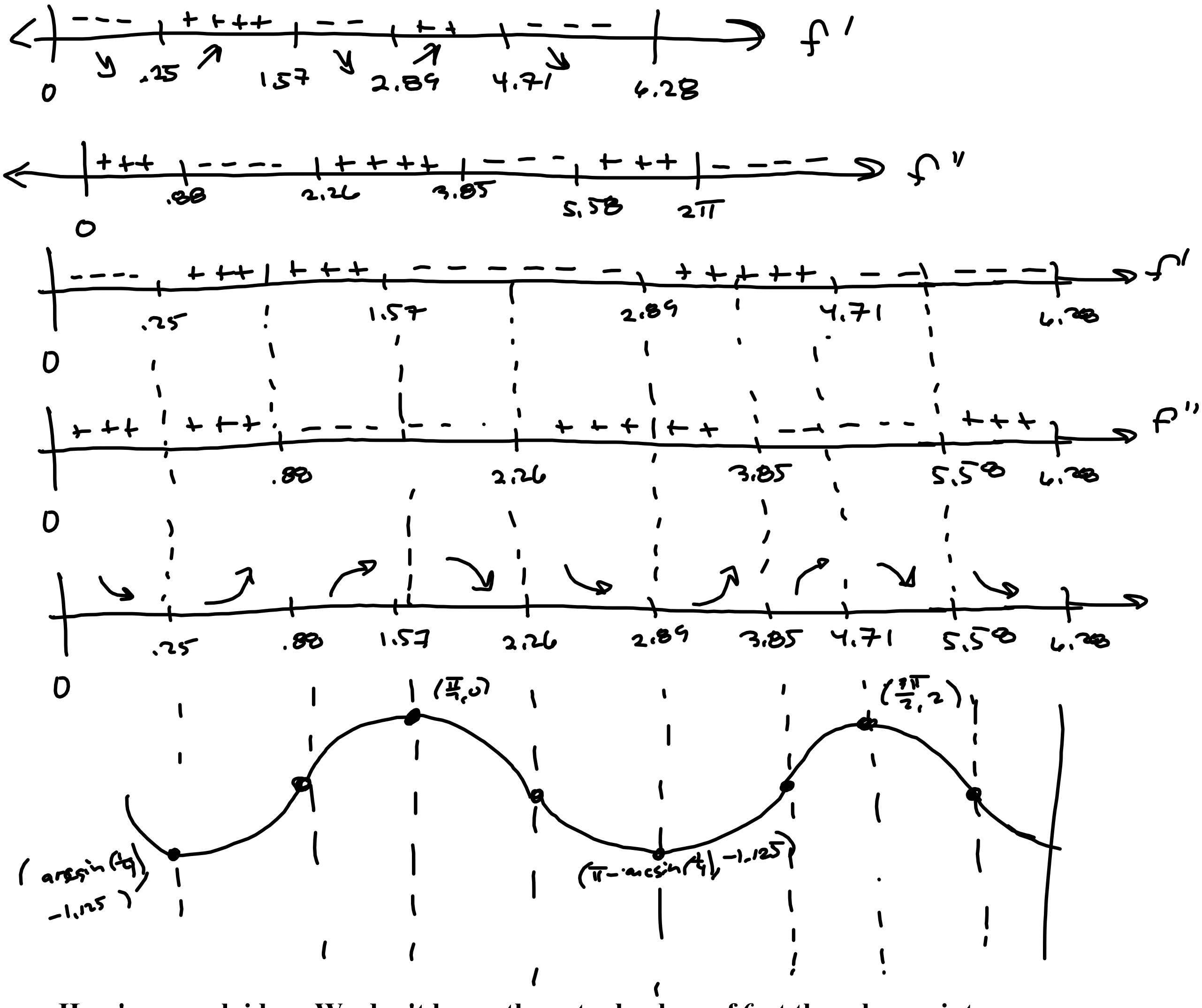
$$\sin(x) = \frac{-1 \pm \sqrt{129}}{8}$$

$$\sin(x) = \frac{-1-\sqrt{129}}{8}, \quad \frac{-1+\sqrt{129}}{8}$$

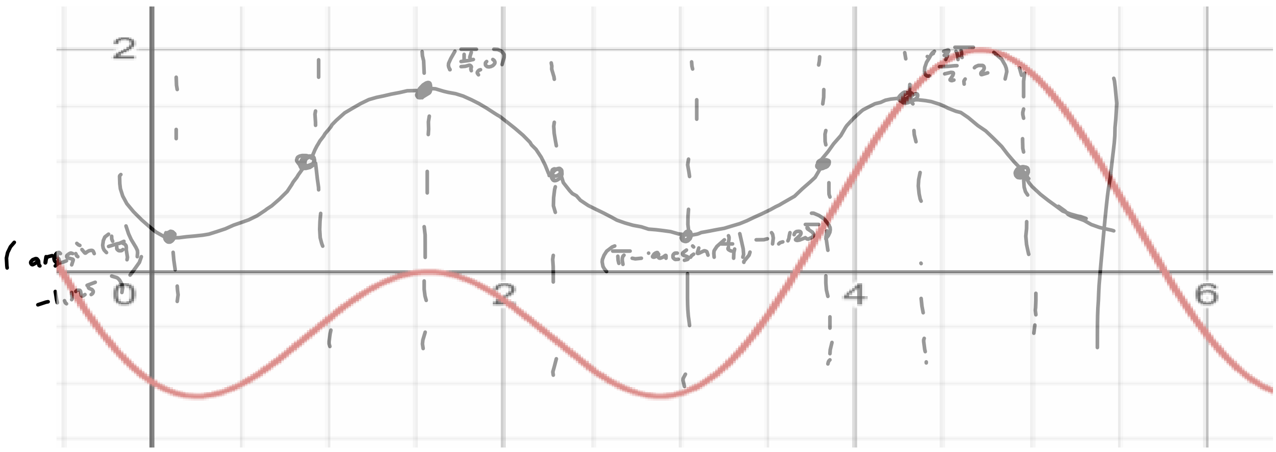


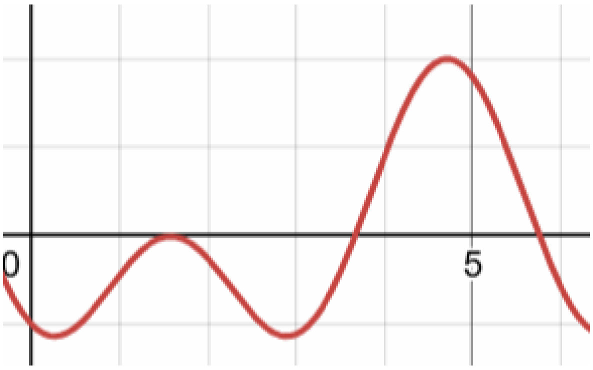


Test:	f''
.5	2.6
1	-.82
2.5	1.43
4	-1.34



Here's a rough idea. We don't know the actual values of f at these key points.





Actual Graph

Limits at Infinity

$$\lim_{x \to \infty} \frac{1-3x}{\sqrt{x^2+1}}$$

Trick: Divide by highest power of x.

$$\frac{1-3x}{\sqrt{x^2+1}} = \frac{x\left(\frac{1}{x}-3\right)}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}} = \frac{x\left(\frac{1}{x}-3\right)}{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}} = \frac{x\left(\frac{1}{x}-3\right)}{|x|\sqrt{1+\frac{1}{x^2}}}$$

Because $x \to \infty$, we know $x > 0 \Longrightarrow |x| = x$

$$\frac{x\left(\frac{1}{x}-3\right)}{x\sqrt{1+\frac{1}{x^2}}} = \frac{\overbrace{\frac{1}{x}}^{\to 0}-3}{1+\underbrace{\frac{1}{x^2}}_{\to 0}} \xrightarrow{x \to +\infty} \frac{-3}{1} = -3$$

$(x \neq 0)$

$\lim_{x \to \infty} f(x) = L$

means "eventually, $f(x)$ is close to L"

which means "Given any $\varepsilon > 0$, I can find an N such that $|f(x) - L| <$ $\forall$ $x > N$.

Example.

$$f(x) = \frac{1-3x}{\sqrt{x^2+1}}.$$

We showed $\lim_{x \to \infty} f(x) = -3$.

Find the SMALLEST $N \ni |f(x) - L| < \varepsilon = 0.1$

Want

$$\left| \frac{1-3x}{\sqrt{x^2+1}} - (-3) \right| < .1$$

$$\Leftrightarrow \frac{1-3x}{\sqrt{x^2+1}} + \frac{3}{1}\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} = \frac{3\sqrt{x^2+1} - 3x + 1}{\sqrt{x^2+1}} \leq 0.1$$

$$\Leftrightarrow 3\sqrt{x^2+1} - 3x + 1 < 0.1\sqrt{x^2+1}$$

$$\Leftrightarrow 2.9\sqrt{x^2+1} - 3x + 1 < 0$$

$$\Rightarrow 2.9\sqrt{x^2+1} < 3x - 1$$

$$(2.9)^2 (x^2+1) < 9x^2 - 6x + 1$$

$$\Rightarrow 2.9^2 x^2 + (2.9^2)(1) < 9x^2 - 6x + 1$$

$$\Rightarrow 8.41x^2 - 9x^2 + 8.41 + 6x - 1 < 0$$

$$\Rightarrow -.59x^2 + 6x + 7.41 < 0$$

$$\Rightarrow .59x^2 - 6x - 7.41 > 0$$

$$\Rightarrow 59x^2 - 600x + 741 > 0$$

$$\begin{array}{r} 2.9 \\ \times\, 2.9 \\ \hline 261 \\ 580 \\ \hline 841 \end{array}$$

$$600^2 - 4(59)(741) =$$

$$360000 - (236)(741)$$

$$= 360000 - 174876$$

$$\begin{array}{r} 360000 \\ -\, 174876 \\ \hline 185124 \end{array}$$

$$\begin{array}{r} 741 \\ \times\, 236 \\ \hline 4446 \\ 22230 \\ 148200 \\ \hline 174876 \end{array}$$

$$x = \frac{600 \pm \sqrt{185124}}{2(59)}$$

$$\Rightarrow |f(x) - L| \ \forall \ x > N = \frac{600 + \sqrt{185124}}{118}$$