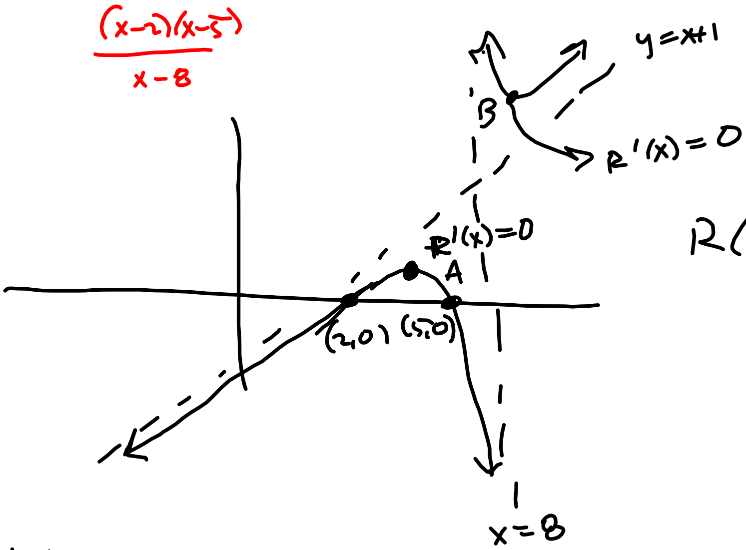
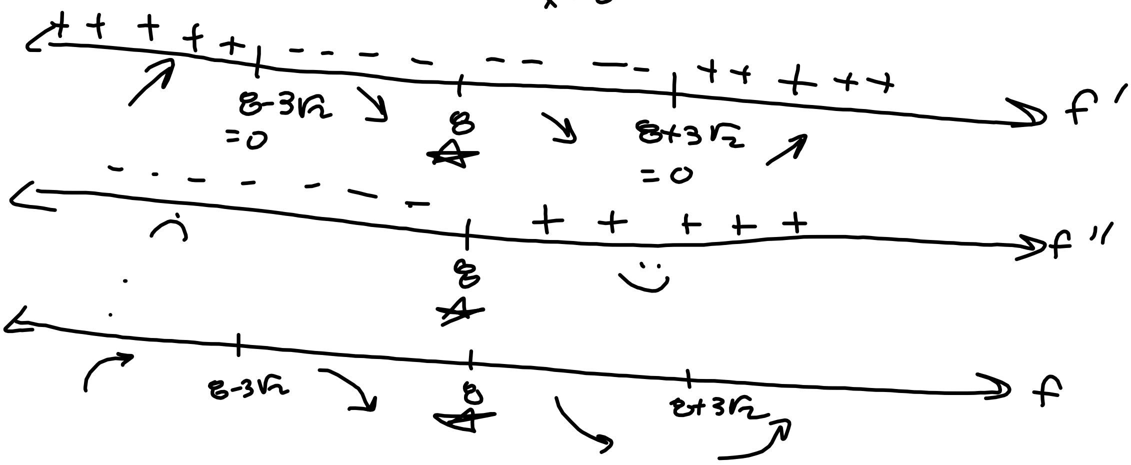
We finish off the graph of the Rational Function from Tuesday, 3/12, using Chapter 3 concepts.

$$\frac{(x-2)(x-5)}{x-8}$$



$$R(x) = \frac{(x-2)(x-5)}{x-8}$$

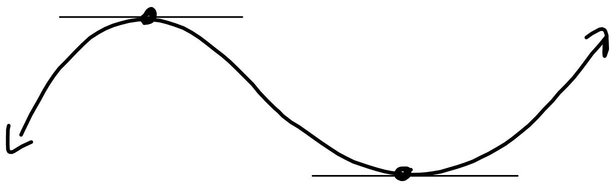


$(3.75735931288, 0.514718625761) \approx (8-3\sqrt{2}, R(8-3\sqrt{2})) = A$

$(12.2426406871, 17.4852813742) \approx (8+3\sqrt{2}, R(8+3\sqrt{2})) = B$

CHAPTER 3 STUFF

Fermat's Theorem: If f is difbl and f achieves a local max/min at $x=c$, then $f'(c) = 0$.



Let $f(x)$ be a function defined on some interval I.

The absolute max for $f(x)$, call it M, satisfies

$$M \geq f(x) \text{ for all } x \in I.$$

The absolute min for $f(x)$, call it m, satisfies

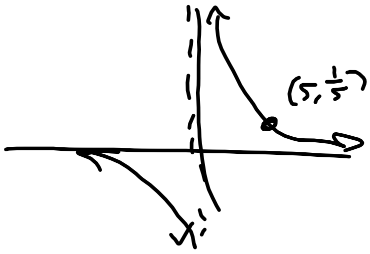
$$m \leq f(x) \quad \forall x \in I.$$

Local max/min $f(\hat{x}) \geq f(x)$ $\forall x$ in some open interval containing $\hat{x}$.

Extreme Value Theorem - If $f(x)$ is cnt^{s} on a closed interval $I = [a,b]$, then f achieves its maximum and minimum on I.

Nonexample

$f(x) = \frac{1}{x}$ on $(0,5)$ is an open interval



No maximum on $(0,5)$

No minimum on $(0,5)$ *

Its greatest lower bound is $y = \frac{1}{5}$, but f never achieves $y = \frac{1}{5}$ & of course, f is unbounded as $x \to 0^+$.

Keys: Continuous & closed interval

STRATEGY: Find zeros of $f'(x)$

Check endpoints $f(a) = ?$, $f(b)?$

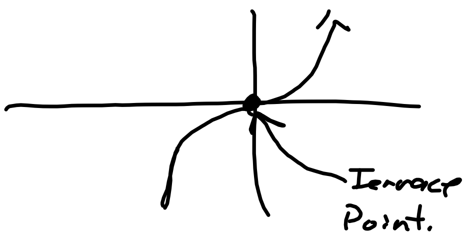
WARNING: $f(c)$ max $\Longrightarrow f'(c) = 0$

$f(c)$ min $\Longrightarrow$ " " "

The converse isn't true, in general, but $f'(x)=0$ is very strongly correlated with max/min values.

$f(x) = x^3$

$f'(x) = 3x^2 = 0$ @ $x = 0$



EXCEPTION You need to be aware of:

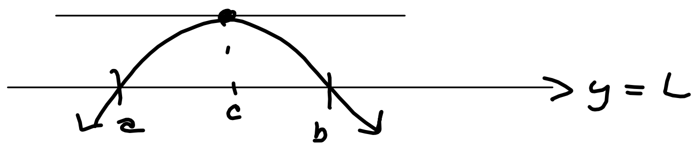
$f'(0)=0$, but $f(0)$ is not an extreme.

Rolle's Theorem:

If $f(x)$ is differentiable for all $x \in (a,b)$ and

" " cnt's for all $x \in [a,b]$, and $f(a) = f(b) = L$

then there is a point $c \in (a,b) \ni f'(c) = 0$



Proof

$\S$ $f(a) = f(b)$.

(i) f is constant. Then $f(x) = L \ \forall \ x \in [a,b]$ & so

$f'(x) = 0 \ \forall \ x \in (a,b)$.

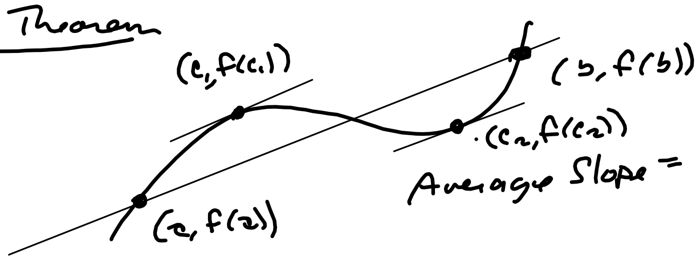
(ii) f is not constant. Then either there's $x \in (a,b) \ni$

$f(x) > L$ or $f(x) < L$ or both.

$f(x) > L$. Then BY E.V.T. f achieves its maximum in (a,b). Then by Fermat's theorem $f'(c) = 0$ at that point. $f'(c) = 0$.

Likewise if $f(x) < L$ anywhere in (a,b).

Mean Value Theorem



Average Slope $= \dfrac{f(b)-f(a)}{b-a}$

f cnt's on $[a,b]$, diff'd on (a,b). Then

$\exists \ c \in (a,b) \ni f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

It says nothing about how to find c.

Find the "c" that is promised for $f(x) = x^3 - 5x^2 + 2x + 1$

on $(a,b) = (0, 2)$.

Check: f is polynomial, so cnt^s & dif'bl everywhere,

so MVT hypotheses are satisfied.

$f(0) = 1$

$f(2) = 2^3 - 5(2)^2 + 2(2) + 1 = 8 - 20 + 4 + 1 = -7$

$(0,1), (2,-7)$.

$(a, f(a)) = (0,1) = (x_1, y_1)$
$(b, f(b)) = (2,-7) = (x_2, y_2)$ $\longrightarrow m_{AVG} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-1}{2-0} = -\frac{8}{2} = -4$

FIND c:

$f'(x) = 3x^2 - 10x + 2 \overset{\text{SET}}{=} -4$

$\longrightarrow 3x^2 - 10x = -6$

$\Longrightarrow 3\left(x^2 - \frac{10}{3}x + \left(\frac{5}{3}\right)^2\right) = -\frac{6}{1} + (3)\left(\frac{25}{9}\right) = \frac{25}{3} - \frac{6}{1} \cdot \frac{3}{3} = \frac{7}{3}$

$\Longrightarrow 3\left(x - \frac{5}{3}\right)^2 = \frac{7}{3}$

$\left(x - \frac{5}{3}\right)^2 = \frac{7}{9}$

$x - \frac{5}{3} = \pm \frac{\sqrt{7}}{3}$

$x = \frac{5 \pm \sqrt{7}}{3} \longrightarrow \boxed{c = \frac{5 - \sqrt{7}}{3}}$