Questions?

Intermediate-Value Theorem Question

$$f := x \rightarrow expand((x-5)\cdot(x+3)\cdot(x-11)\cdot(x+6))$$

$$f(x) = x^4 - 7x^3 - 71x^2 + 207x + 990$$

$+ \quad -6 \quad - \quad -3 \quad + \quad 5 \quad - \quad 11 \quad +$

Show that f(x) has a zero in the interval (4, 7) (or $(-5, 12)$)

$f(4)$:

$$\begin{array}{r|rrrrr} 4 & 1 & -7 & -71 & 207 & 990 \\ & & 4 & -12 & -332 & -500 \\ \hline & 1 & -3 & -83 & -125 & \text{Positive} \end{array}$$

$$\begin{array}{r} 83 \\ \times\ 4 \\ \hline 332 \end{array}$$

$f(7)$

$$\begin{array}{r|rrrrr} 7 & 1 & -7 & -71 & 207 & 990 \\ & & 7 & 0 & -497 & -2030 \\ \hline & 1 & 0 & -71 & -290 & \text{Negative} \end{array}$$

$$\begin{array}{r} 290 \\ \times\ 7 \\ \hline 2030 \end{array} \qquad \begin{array}{r} 79 \\ \times\ 6 \\ \hline 474 \end{array} \qquad \begin{array}{r} 267 \\ \times\ 6 \\ \hline 1602 \end{array} \qquad \begin{array}{r} 71 \\ \times\ 7 \\ \hline 497 \end{array}$$

This shows $f(7) < 0$ and $f(4) > 0$. f is a polynomial and hence cnts on $(-\infty, \infty) \Rightarrow \exists\, c \in (4, 7)$ $\ni f(c) = 0$ by IVT.

2. Consider the piecewise-defined function $f(x) = \begin{cases} x^2 - 4x + 5 & \text{if } x < 3 \\ -\frac{1}{2}x + \frac{7}{2} & \text{if } x \geq 3 \end{cases}$.

a. (5 pts) Sketch the graph of $f(x)$. Label the x- and y-intercepts, the suture point(s), and the vertex of the quadratic piece, if it's in the picture. When I say "Label," I mean an ordered pair, like (0, 5), next to the point

$x^2+4x+5 \qquad x<3$

$= (x^2+4x+2^2 - 4 + 5$

$= (x+2)^2 + 1 \overset{\text{SET}}{=} 0 \Rightarrow (x+2)^2 = -1$ No Real Sol'n.

$\boxed{(h,k) = (-2,1)}$ $\qquad f(0) = 5 \rightarrow \boxed{(0,5)}$

$\lim_{x\to 3^-} \quad 3^2+4(3)+5 = 26 \rightarrow \boxed{(3,26) \text{ SUTURE}} = \lim_{x\to 3^-} f(x)$

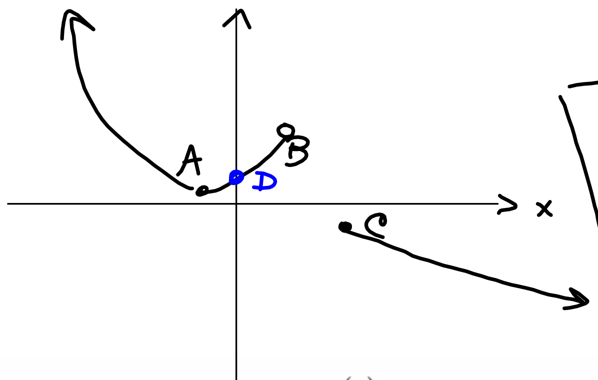
$y = -\frac{1}{2}x + \frac{7}{2}$

x	y
0	7/2
7	0

$\frac{1}{2}x = \frac{7}{2} \Rightarrow x = 7$

$-\frac{1}{2}(3) + \frac{7}{2} = -\frac{3}{2} + \frac{7}{2} = -\frac{4}{2} = -2$

$\boxed{(3,-2) = f(3)}$



A = (−2,1)
B = (3,26)
C = (3,−2)

D = (0,5) FORGOT!

b. (5 pts) On what interval(s) is $f(x)$ continuous? Explain.

f's cts on $(-\infty,3) \cup (3,\infty)$

Technically we'd say cts @ x=3 from the right. Not cts @ x=3 b/c of a jump discontinuity from y=26 to y=−2

Graph $3x^2 - 5x - 11$

$= 3\left(x^2 - \frac{5}{3}x + \left(\frac{5}{6}\right)^2\right) - \frac{11}{1} \quad -3\left(\frac{25}{36}\right)$

$= \underline{3\left(x - \frac{5}{6}\right)^2 - \frac{157}{12}}$ set 0 $\rightarrow$

$(h,k) = \left(\frac{5}{6}, -\frac{157}{12}\right)$

$3\left(x - \frac{5}{6}\right)^2 = \frac{157}{12}$

$\left(x - \frac{5}{6}\right)^2 = \frac{157}{36}$

$x - \frac{5}{6} = \pm\sqrt{\frac{157}{36}} = \pm\frac{\sqrt{157}}{6}$

$x = \frac{5 \pm \sqrt{157}}{6}$

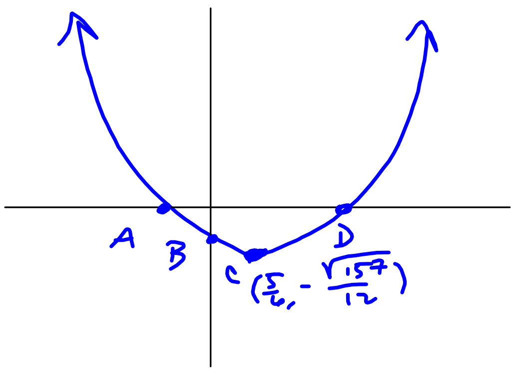
scratch:

$-\frac{11}{1}\cdot\frac{12}{12} - \frac{25}{12}$

$= \frac{-132-25}{12}$

$= -\frac{157}{12}$

$\lfloor 157$



$\cdot\,(0,-11)$ y-int

$A = \left(\frac{5-\sqrt{157}}{6}, 0\right)$

$B = (0,-11)$

$C = \left(\frac{5}{6}, -\frac{157}{12}\right)$

$D = \left(\frac{5+\sqrt{157}}{6}, 0\right)$

$f(x) = x^2 - 4x + 5$

Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Find $f'(x)$ by the limit definition

Find $f'(x)$ by taking the limit of the difference quotient.

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 \cancel{-4x} - 4h \cancel{+5} - \cancel{x^2} \cancel{+4x} \cancel{-5}}{h}$$

$$f(x+h) = (x+h)^2 - 4(x+h) + 5$$
$$= x^2 + 2xh + h^2 - 4x - 4h + 5$$
$$-f(x) = -(x^2 - 4x + 5)$$
$$= -x^2 + 4x - 5$$

$$= \frac{2xh + h^2 - 4h}{h} = \frac{h(2x + h - 4)}{h}$$

$$\xrightarrow{h \to 0} \boxed{2x - 4.}$$

4. The point $P(2,-4)$ lies on the graph of $f(x)=x^2-2x-4$.

a. (5 pts) Write the equation of the tangent line to $f(x)$ at P.

b. (5 pts) Sketch a graph of $f(x)$ and the tangent line to $f(x)$ at the point P.

4a

$f'(x) = 2x-2$

$f'(2) = 2(2)-2 = \boxed{2 = f'(2) = } f'(x_1)$

$(2,-4) = (x_1, y_1)$

$= (x_1, f(x_1))$

$y = m(x-x_1)+y_1$

$L(x) = f'(x_1)(x-x_1)+f(x_1)$

$= \boxed{2(x-2)-4 = L(x)}$

SET $\underline{=} 0 \Longrightarrow 2(x-2)=4$

$x-2=2$

$x=4$

$x=0 \Longrightarrow$

$2(-2)-4 = -8$

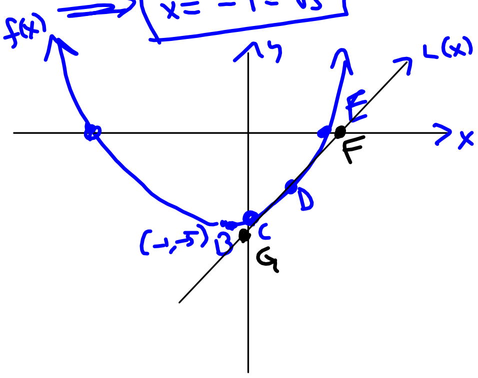
b) Graph:

$f(x) = x^2-2x-4 = x^2-2x+1^2-1-4$

$= (x+1)^2-5$ SET $\underline{=} 0$

$(h,k) = (-1,-5)$

$f(x) \Longrightarrow \boxed{x = -1 \pm \sqrt{5}}$



$A = (-1-\sqrt{5}, 0)$

$B = (-1,-5)$

$C = (0,-4)$

$D = (2,-4)$

$E = (-1+\sqrt{5}, 0)$

$F = (4,0)$

$G = (0,-8)$