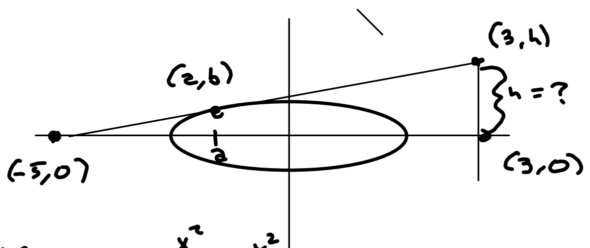


$x^2+4y^2=5$ $\qquad \frac{x^2}{5}+\frac{y^2}{(\frac{5}{4})}=1$

Bradley helped on Wednesday

$m = \frac{h}{8} = \frac{b}{|-5-a|} = \frac{b}{a+5}$

$2x+8yy'=0$

$y' = -\frac{x}{4y}$

$\frac{b}{a+5} = -\frac{a}{4b} \implies$

$$\boxed{\begin{gathered} 4b^2 = -a^2-5a \\ a^2+4b^2 = -5a = 5 \\ a=-1 \end{gathered}}$$

$a=-1$

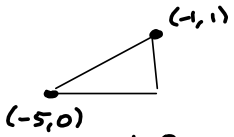
$a^2+4b^2=5$

$1+4b^2=5$

$4b^2=4$

$b^2=1$

$b=\pm 1 \implies b=1$



$m = \frac{1-0}{-1-(-5)} = \frac{1}{-1+5} = \frac{1}{4} = m = \frac{h}{8}$

$\implies \boxed{h=2}$

$f(x) = \sqrt{x+2}$

Find $f'(a)$

Instead of $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$, do this:

$$\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$\frac{f(x)-f(a)}{x-a} = \left(\frac{\sqrt{x+2}-\sqrt{a+2}}{x-a}\right)\left(\frac{\sqrt{x+2}+\sqrt{a+2}}{\sqrt{x+2}+\sqrt{a+2}}\right) = \frac{x+2-(x+a)}{x-a}$$

$$= \frac{\cancel{(x-a)}}{\cancel{(x-a)}(\sqrt{x+2}+\sqrt{a+2})} \xrightarrow{x \to a} \frac{1}{\sqrt{a+2}+\sqrt{a+2}} = \boxed{\frac{1}{2\sqrt{a+2}} = f'(a)}$$

$$= \frac{1}{\sqrt{x+2}+\sqrt{a+2}} \quad (x \neq a)$$

skipped this step, but showed just enough.

1. (10 pts) Evaluate $\lim_{x\to -7} \dfrac{x^2+10x+21}{x^2-5x-84}$ by factoring and simplifying.

$x^2+10x+21 = (x+3)(x+7)$

$x^2-5x-84 = (x-12)(x+7)$

$\dfrac{x^2+10x+21}{x^2-5x-84} = \dfrac{(x+3)\cancel{(x+7)}}{(x-12)\cancel{(x+7)}} \xrightarrow{x\to -7} \dfrac{-7+3}{-7-12} = \dfrac{-4}{-19} = \dfrac{4}{19}$

2. Evaluate each limit, if it exists. If it does not exist give a good reason.

a. (5 pts) $\lim_{x\to -3^-} \dfrac{x^2-8x-33}{|x+3|}$

Define $\dfrac{x^2-8x-33}{|x+3|} = f(x)$

2a $\lim_{x\to 3} f(x) = \lim f$

$|x+3| = \begin{cases} x+3 & \text{if } x+3 \ge 0 \\ -(x+3) & \text{if } x+3 < 0 \end{cases} = \begin{cases} x+3 & \text{if } x \ge -3 \\ -(x+3) & \text{if } x < -3 \end{cases}$

$x\to 3^- \longrightarrow x < -3$

$\longrightarrow \dfrac{x^2-8x-33}{|x+3|} = \dfrac{(x-11)\cancel{(x+3)}}{-\cancel{(x+3)}} \xrightarrow{x\to 3^-} \dfrac{(3-11)}{-1} = \dfrac{-8}{-1} = \boxed{8 = \lim f}$

-3^- -3 -14 14

Thx, Andrew!

b. (5 pts) $\lim_{x\to -3^+} \dfrac{x^2-8x-33}{|x+3|}$

2b $\lim_{x\to -3^+} f(x) = \lim f$

$x\to -3^+ \longrightarrow x > -3 \longrightarrow$

$\dfrac{x^2-8x-33}{|x+3|} = \dfrac{x-8x-33}{x+3} = \dfrac{(x-11)\cancel{(x+3)}}{\cancel{x+3}} \xrightarrow{x\to -3^+} \dfrac{-3-11}{1} = \boxed{-14 = \lim f}$

c. (5 pts) $\lim_{x\to -3} \dfrac{x^2-8x-33}{|x+3|}$

2c $\lim_{x\to 3} \dfrac{x^2-8x-33}{|x+3|}$ $\nexists$, b/c

$\lim_{x\to -3^-} f(x) = 14 \neq -14 = \lim_{x\to -3^+} f(x)$.

5. (5 pts each) Compute the derivatives of each of the following. Do not simplify your answer.

c. $y = \dfrac{x^2+5x}{7x-1}$

5c

$y = \dfrac{x^2+5x}{7x-1} = \dfrac{f}{g}$

$\Rightarrow y' = \left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$

$= \boxed{\dfrac{(2x+5)(7x-1) - (x^2+5x)(7)}{(7x-1)^2} = y'}$

d. $y = \left(x^2\sin(x)\right)^{-4}\left(x^2+2x\right)^3 = fg$

5d

$y = (x^2\sin(x))^{-4}(x^2+2x)^3 = fg$

$\Rightarrow y' = f'g + fg' = \boxed{-4(x^2\sin(x))^{-5}(2x\sin(x) + x^2\cos(x))(x^2+2x)^3 + (x^2\sin(x))^{-4}(3(x^2+2x)^2(2x+2)) = y'}$

e. $y = \sin\left(\sqrt{\tan\left(x^2+2x\right)}\right)$

5e

$y = \sin\left((\tan(x^2+2x))^{\frac{1}{2}}\right)$

$\Rightarrow \boxed{y' = \cos\left((\tan(x^2+2x))^{\frac{1}{2}}\right)\left(\frac{1}{2}(\tan(x^2+2x))^{-\frac{1}{2}}(\sec^2(x^2+2x))(2x+2)\right)}$

6. (5 pts) Find an equation of the tangent line to $f(x) = \tan(x)$ at $x = \frac{\pi}{4}$. Then sketch the graph of this situation, with the function and its tangent line, together on the same set of axes.

$y = \tan(x) = f(x)$. Tangent line to $f(x)$ @ $x = \frac{\pi}{4}$:

$f'(x) = \sec^2(x)$

$f'(\frac{\pi}{4}) = \sec^2(\frac{\pi}{4})$

$= \frac{1}{\cos^2(\frac{\pi}{4})} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2} = f'(\frac{\pi}{4})$

$f(\frac{\pi}{4}) = \tan\frac{\pi}{4} = 1$

$L(x) = f'(x_1)(x - x_1) + f(x_1)$

$= \boxed{\sqrt{2}\,(x - \frac{\pi}{4}) + 1 = L(x)}$

7. (5 pts) Use your result from the previous problem to approximate $\tan(42^0)$

$L(42^0) = \sqrt{2}\,(42^0 - \frac{\pi}{4}) + 1$

$= \sqrt{2}\,\left((\frac{42^0}{1})(\frac{\pi}{180^0}) - \frac{\pi}{4}\right) + 1$

$\frac{7\pi}{30}$

$= \sqrt{2}\,(\frac{7\pi}{30} - \frac{\pi}{4}) + 1$

$= \sqrt{2}\,(\frac{14\pi}{60} - \frac{15\pi}{60}) + 1$

$= \sqrt{2}\,(-\frac{\pi}{60}) + 1 = \boxed{-\frac{\pi\sqrt{2}}{60} + 1}$

8. (10 pts) Find all values of x such that $f(x) = 2\sin(x)\cos(x) + x$ has a horizontal tangent.

8. $f(x) = 2\sin(x)\cos(x) + x \longrightarrow$

$f'(x) = 2\cos(x)\cos(x) + 2\sin(x)(-\sin(x)) + 1$

$= 2\cos^2(x) - 2\sin^2(x) + 1 \overset{\text{SET}}{=} 0$

$\Longrightarrow 2(1 - \sin^2(x)) - 2\sin^2(x) = -1$

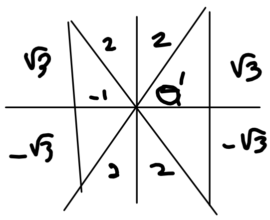
$\Longrightarrow 2 - 2\sin^2(x) - 2\sin^2(x) = -1$

$\Longrightarrow -4\sin^2(x) = -3$

$\Longrightarrow 4\sin^2(x) = 3$

$\Longrightarrow \sin^2(x) = \frac{3}{4}$

$\Longrightarrow \sin(x) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$



$\theta' = 60^\circ = \frac{\pi}{3}$ = reference angle

$x = \frac{\pi}{3}, \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

ALL SOLUTIONS:

$$\boxed{\frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in \mathbb{Z}}$$

OR

$$\boxed{\frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi, n \in \mathbb{Z}}$$

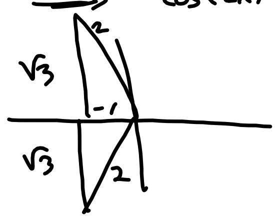
Either answer. Don't need both.

(8) $f(x) = 2\sin(x)\cos(x) + x$

$= \sin(2x) + x$

$\Rightarrow f'(x) = 2\cos(2x) + 1 \overset{\text{set}}{=} 0$

$\Rightarrow \cos(2x) = -\frac{1}{2}$



$2x = \frac{2\pi}{3}, \frac{4\pi}{3} \Rightarrow$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

we need to find all $x \in [0, 2\pi)$

$x \in [0, 2\pi) \Rightarrow$

$2x \in [0, 4\pi)$

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3} + 2\pi, \frac{4\pi}{3} + 2\pi$

$= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$