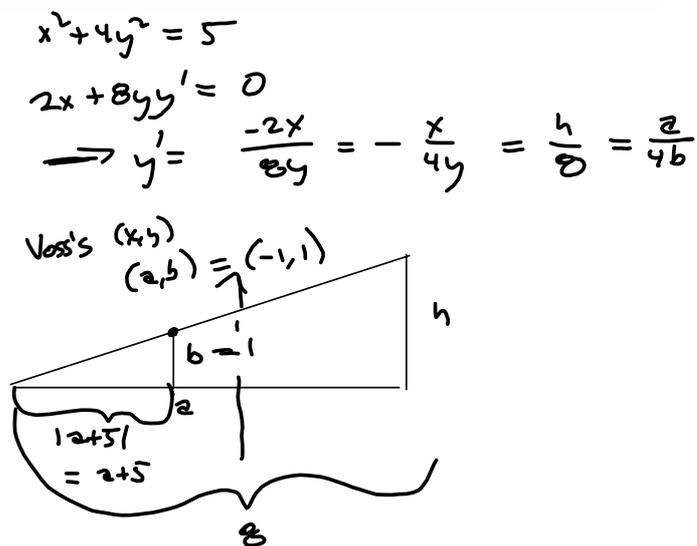


4. (10 pts) The figure shows a lamp located three units to the right of the y-axis and a shadow created by the elliptical region  $x^2 + 4y^2 \leq 5$ . If the point  $(-5, 0)$  is on the edge of the shadow, how far above the x-axis is the lamp located? Do the best writeup possible for this exercise.



$$m_{tan} = \frac{h}{8} = \frac{b}{2+5} =$$

$(a,b)$  is on the ellipse  $\rightarrow$

$(a,b)$  is on the line

$$y = \frac{h}{8}(x+5)$$

$$b = \frac{h}{8}(2+5)$$

$$8b = 2h + 5h$$

$$a^2 + 4b^2 = 5$$

$$y' = -\frac{x}{4y} = \frac{y}{x+5} \rightarrow$$

$$-x^2 - 5x = 4y^2$$

$$\rightarrow x^2 + 4y^2 = -5x = 5$$

Bradley's  $x = -1 = a$  min

$$x^2 + 4y^2 = 5$$

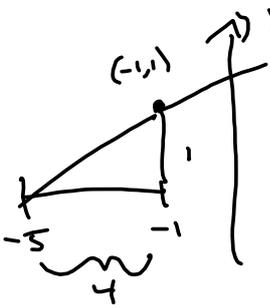
$$(a,b) = (-1, 1)$$

$$(-1)^2 + 4y^2 = 5$$

$$4y^2 = 4$$

$$y^2 = 1$$

$$y = \pm 1 = \boxed{1=y}$$



$$m = \frac{1}{4} = \frac{h}{8} \rightarrow h = \frac{8}{4} = 2$$

## #6 Section 2.9

Verify the given linear approximation at  $a = 0$ . Then use a graphing calculator or computer to determine the values of  $x$  for which the linear approximation is accurate to within 0.1. (Round your answers to three decimal places. Enter your answer using interval notation.)

$$(1 + 4x)^{-4} \approx 1 - 16x$$

$$f(x) = (4x+1)^{-4}$$

$$f(0) = 1^{-4} = 1 \rightarrow (x_1, y_1) = (x_1, f(x_1)) = (0, 1)$$

$$f'(x) = -4(4x+1)^{-5}(4)$$

$$f'(0) = -4(1)^{-5}(4) = -16 \leftarrow f'(0) = m_{t_{a,a}}$$

$$L(x) = f'(x_1)(x - x_1) + f(x_1)$$

$$= -16(x - 0) + 1$$

$$= -16x + 1$$

$$y = -16x + 1$$

Now, we want to know the range of  $x$ -values for which  $|f(x) - L(x)| < 0.1$

$$\text{i.e., we want } |(4x+1)^{-4} - (-16x+1)| < 0.1$$

$$\Rightarrow -0.1 < (4x+1)^{-4} + 16x - 1 < 0.1$$

$$x \in (-0.02268, 0.027695) \approx (-0.023, 0.028)$$

$$|f(x) - L(x)| < 0.1$$

I rounded up!  
Not good!

Correct answer (Don't round up, Steve!)

$$x \in (-0.022, 0.027) \rightarrow$$

$$|f(x) - L(x)| < 0.1$$

Book did:

$$(4x+1)^{-4} - 0.1 < -16x + 1 < (4x+1)^{-4} + 0.1$$

Difference is 0.1

<https://www.wolframalpha.com/input?i=solve%28%284x%2B1%29%5E%28-4%29%2B16x-1%3D0.1>



Difference is -0.1

$$-0.02769$$

We did this with Desmos

$$|(4x+1)^{-4} - (-16x+1)| < 0.1$$

$$\Rightarrow (4x+1)^{-4} + 16x - 1 = 0.1$$

$$(4x+1)^{-4} + 16x - 1 = -0.1$$

$$(4x+1)^{-4} = -16x + 1.1$$

$$(4x+1)^{-4} = -16x + 0.9$$