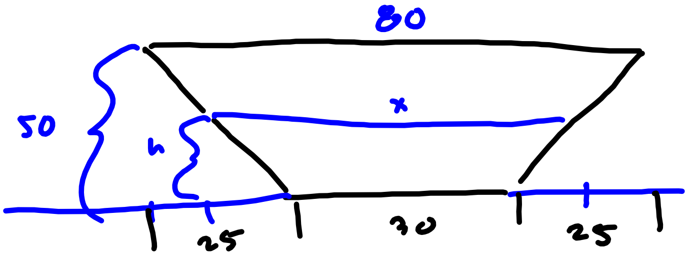
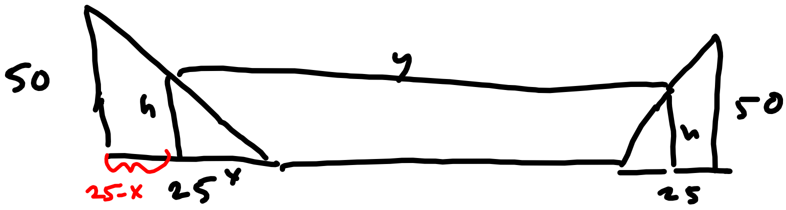




$$\frac{50}{25} = \frac{h}{x}$$

$$y = 80 - 2(25-x)$$
$$= 80 - 50 + 2x$$
$$= 2x + 30$$
$$= 2\left(\frac{h}{2}\right) + 30$$
$$= h + 30$$

$$\frac{h}{x} = \frac{50}{25}$$

$$h = 2x$$
$$x = \frac{h}{2}$$

$$\frac{1}{2}(30 + y)h \cdot 1000$$
$$= 500(y+30)h$$
$$= 500(h+30+30)h$$
$$= 500(h+60)h$$

$$\frac{dV}{dt} = .2\,m^3/min = 500[h'h + (h+60)h']$$
$$= 500[h + h + 60]h' = \left(\frac{.2\,m^3}{min}\right)\left(\frac{100^3\,cm^3}{m^3}\right)$$
$$= 500[2h+60]h' = \left(\frac{2}{10}\right)(100)^3$$

Set $h=30$ $= 500[2(30)+60]h' = 500[120]h' = (2\times10^{-1})(10^2)^3 = (2\times10^{-1})(10^6)$

60000 $\leftarrow$ $\rightarrow$ 6000 $h' =$ $= 2\times10^5$

$$\rightarrow h' = \frac{200000}{60000} = \frac{200}{6}$$ NO

$h' = \frac{10}{3}\,\frac{cm}{min}$

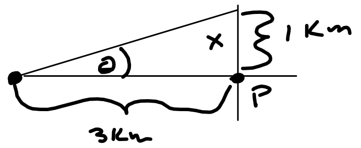
$$\frac{200000}{6000}$$

$$\frac{12000}{2} = 6000$$

$h' = \frac{100}{3}\ \frac{cm}{min}$ Ans is $\frac{10}{3}$

From 10^{th} Edition: 2.8 #44

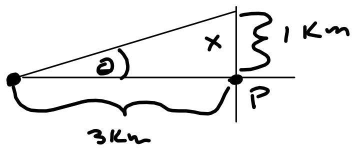
A lighthouse is 3 Km from shore. Call the nearest point P. Light makes 4 revolutions / minute. How fast is the beam moving along shoreline when it is 1 Km from P



Let θ = angle beam makes with a line from lighthouse to point P. (radians)

Let x = distance from P to where beam hits the shore-line (Km)

Given light revolves 4 times per minute. We need to convert that to angular speed $\left(\frac{d\theta}{dt}\right)$

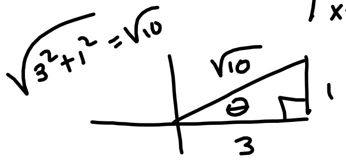


$$\left(\frac{4 \text{ rev}}{\text{min}}\right)\left(\frac{2\pi \text{ radians}}{1 \text{ rev}}\right) = 8\pi \quad \left(\frac{\text{radians}}{\text{minute}}\right)$$

$$\frac{x}{3} = \tan\theta \; !$$

$$\longrightarrow \frac{1}{3}\frac{dx}{dt} = (\sec^2\theta)\left(\frac{d\theta}{dt}\right)$$

Want $\left.\frac{dx}{dt}\right|_{x=1}$. We need $\left.\theta\right|_{x=1}$



$$\theta = \arctan\left(\frac{1}{3}\right)$$

$$\longrightarrow \frac{dx}{dt} = 3\left(\sec^2\left(\arctan\left(\frac{1}{3}\right)\right)\right) \cdot 8\pi$$

$$= 3\left(\frac{\sqrt{10}}{3}\right)^2 \cdot 8\pi$$

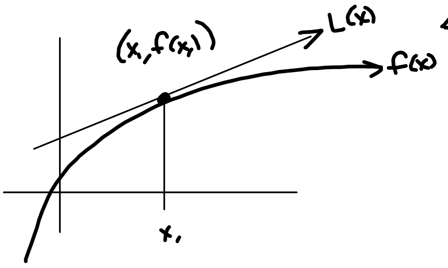
$$= 3\left(\frac{10}{9}\right)(8\pi) = \frac{(10)(8)\pi}{3} = \boxed{\frac{80\pi}{3} \ \frac{\text{Km}}{\text{min}}}$$

$$\approx 83.7758040957 \ \frac{\text{Km}}{\text{min}}$$

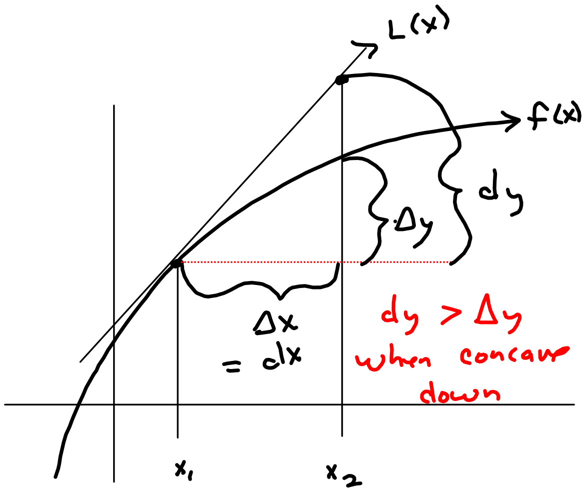
§ 2.9 Linear Approximations and Differentials

Recall: $f(x)$ is differentiable on a neighborhood (open line segment) containing $x = x_1$. Then the LINEARIZATION $L(x)$ of $f(x)$ @ $x = x_1$ is

$$L(x) = L_{x_1}(x) = f'(x_1)\underbrace{(x - x_1)}_{\Delta x} + f(x_1)$$



$L(x) \approx f(x)$ close to $x = x_1$



$$\frac{dy}{dx} = f'(x) \longrightarrow$$

$$dy = f'(x)\,dx$$

$$\Delta y \approx f'(x)\,dx = f'(x)\,\Delta x$$

Also

$$f(x_2) - f(x_1) \approx f'(x_1)\,\Delta x$$

$$\longrightarrow f(x_2) \approx f'(x_1)\,\Delta x + f(x_1)$$
$$= f'(x_1)(x_2 - x_1) + f(x_1)$$

Error Estimates (Like #1 on week 03 written)

$$\Delta y \approx f'(x_1)\,\Delta x$$

$$f(x) = x^3$$

Want $y = 30000 \pm 10$,

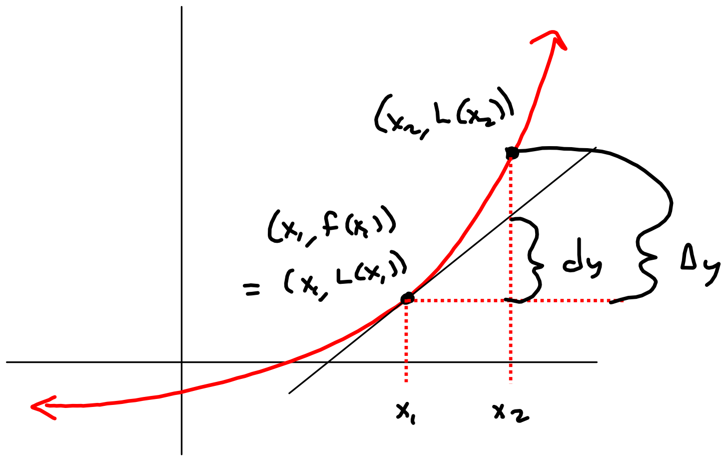
Want $\Delta y \le 10$

$$\Delta y \approx 3x_1^2\,\Delta x$$

$$10 = 3\left(\sqrt[3]{30000}^{\,2}\right)\Delta x \Longrightarrow$$

$$\Delta x = \delta = \frac{10}{3\sqrt[3]{30000}^{\,2}} \approx 0.00345248056217$$

Answer was $.00345$
$\approx .0035$



$dy < \Delta y$ when concave up!

Approximate $\sqrt{99}$ using a differential.

$$f(99) \approx \underbrace{f'(x_1)(99 - x_1)}_{dy \approx \Delta y} + f(x_1) = f'(x)\Delta x + f(x_1)$$

$$= f'(x)dx + f(x_1)$$

$x_1 = 100$ is nice!

$$= f'(100)(99 - 100) + \sqrt{100}$$

$$= f'(100)\boxed{(-1)} + 10 \qquad \Delta x = dx$$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{2(10)} = \frac{1}{20}$$

$$= \frac{1}{20}(-1) + 10$$

$$= 10 - \frac{1}{20} = \frac{200 - 1}{20} = \frac{199}{20}$$

$\sqrt{99} \approx$ 9.94987437107 calculator

$\sqrt{99} \approx$ 9.94987437107 Linear Approx.

Approximate $\sin(31^\circ)$

$x_1 = 30^\circ$ is nice

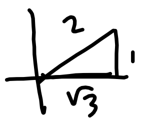
For <u>ANY</u> of this "calculus stuff" to work, we <u>HAVE</u> to be in radians

Why? Recall : Proof of $\frac{d}{dx}[\sin(x)] = \cos(x)$

Depended on θ = arc length on the unit circle.

$$f'(x) = \cos(x)$$

$$x_1 = 30^\circ = \frac{\pi}{6} \longrightarrow$$



$$L(x) = \cos\left(\frac{\pi}{6}\right)\left(x - \frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right)$$

$$= \frac{\sqrt{3}}{2}\left((31^\circ)\left(\frac{\pi}{180^\circ}\right) - \frac{\pi}{6}\right) + \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}\left(\frac{31\pi}{180} - \frac{30\pi}{180}\right) + \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}\left(\frac{\pi}{180}\right) + \frac{1}{2}$$

$$= \frac{\sqrt{3}\,\pi}{360} + \frac{1}{2}$$

$f(31^\circ) \approx 0.51503807491$ Desmos

$L(31^\circ) \approx 0.515114994702 \approx f(31^\circ)$