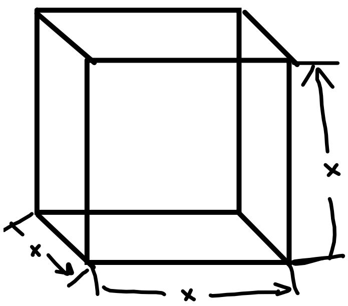
If V is the volume of a cube with edge length x and the cube expands as time passes, find dV/dt in terms of dx/dt.

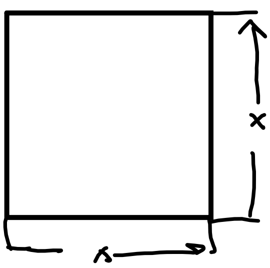


Let x = edge length of a cube
V = volume of a cube

$V = x^3$. Find $\frac{dV}{dt}$ in terms of $\frac{dx}{dt}$. Then

$$\boxed{\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}}$$

Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2?



x = length of a side of a square in cm.

A = Area of square in cm^2

Given $\frac{dy}{dt} = 6 \frac{cm}{s}$.

Find $\left.\frac{dA}{dt}\right|_{A=16\ cm^2}$

$A = x^2 \longrightarrow$

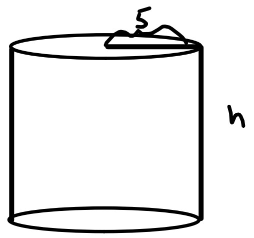
$\frac{dA}{dt} = 2x \cdot \frac{dx}{dt} \longrightarrow$

$$\left.\frac{dA}{dt}\right|_{A=16} = 2(4)(6)\ cm^2/s = \boxed{72\ \frac{cm^2}{s} = \left.\frac{dA}{dt}\right|_{A=16}}$$

A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing?

$\frac{dV}{dt} = 3 \frac{m^3}{min}$

A cylindrical tank of radius 5 m is filling with H_2O at a rate of $3 \frac{m^3}{min}$. How fast is the height of the water increasing?



h = height of cylinder in m.

V = volume " " " m^3

$V = (\text{area of base})(\text{height})$

$= (\pi r^2)(h)$

$= \pi r^2 h$, where $r = 5$ m is radius.

Given $\frac{dV}{dt} = 3 \frac{m^3}{min}$, find $\frac{dh}{dt}$

$V = \pi (5)^2 (h) = 25\pi h \longrightarrow$

$\frac{dV}{dt} = 25\pi \frac{dh}{dt} \longrightarrow$

$\left(\frac{1}{25\pi}\right)\left(\frac{dV}{dt}\right) = \frac{dh}{dt} \longrightarrow$

$\left(\frac{1}{25\pi}\right)(3) = \boxed{\frac{3}{25\pi} \frac{m}{min} = \frac{dh}{dt}}$

This is much more interesting if r also varies.

$V = \pi r^2 h \longrightarrow$

$\frac{dV}{dt} = \pi \left[2r \frac{dr}{dt} h + r^2 \frac{dh}{dt} \right]$, via product rule & chain rule.

Suppose $y = \sqrt{2x + 1}$, where x and y are functions of t.

(a) If $dx/dt = 3$, find dy/dt when $x = 4$.

$y = (2x+1)^{\frac{1}{2}} \Longrightarrow$

$\frac{dy}{dt} = \underbrace{\frac{1}{2}(2x+1)^{-\frac{1}{2}}\left(2\frac{dx}{dt}\right)}_{\text{chain Rule on } (2x+1)^{\frac{1}{2}}}$

Want $\left.\frac{dy}{dt}\right|_{\substack{x=4 \\ \frac{dx}{dt}=3}} = \frac{1}{2}(2(4)+1)^{-\frac{1}{2}}(2\cdot 3)$

$= \frac{1}{2}(9)^{-\frac{1}{2}}(6) = 3\left(\frac{1}{\sqrt{9}}\right) = 3\left(\frac{1}{3}\right) = \boxed{1 = \left.\frac{dy}{dt}\right|_{\substack{x=4 \\ x'=3}}}$

(b) If $dy/dt = 5$, find dx/dt when $x = 12$.

$\frac{dy}{dt} = \frac{1}{2}(2x+1)^{-\frac{1}{2}}\left(2\frac{dx}{dt}\right)$

$\frac{dy}{dt} = 5$, $x = 12$, and we want

$\left.\frac{dx}{dt}\right|_{\substack{y'=5 \\ x=12}}$: $5 = \frac{1}{2}(2(12)+1)^{-\frac{1}{2}}(12)\frac{dx}{dt}$

$5 = 6(25)^{-\frac{1}{2}}\frac{dx}{dt} = 6\left(\frac{1}{5}\right)\frac{dy}{dt}$

$\Longrightarrow \frac{dx}{dt} = 5\left(\frac{5}{6}\right) = \boxed{\frac{25}{6} = \frac{dx}{dt}}\Bigg|_{\substack{y'=5 \\ x=12}}$

If $x^2 + y^2 + z^2 = 9$, $dx/dt = 5$, and $dy/dt = 4$, find dz/dt when $(x, y, z) = (2, 2, 1)$.

$x^2+y^2+z^2=9 \longrightarrow$ $\quad x' = \frac{dx}{dt}$

$2xx'+2yy'+2zz'=0$ $\quad y' = \frac{dy}{dt}$

The rest is just plugging in and finding the desired quantity. $\quad z' = \frac{dz}{dt}$

More Rational Function Review

We've done one that's proper:

$$R(x) = \frac{a_n x^n + \cdots + a_0}{b_{n+m} x^{n+m} + \cdots + b_0} \xrightarrow{x \to \infty} 0 = \text{H.A.}$$

Two more possibilities:

1. Tie: $\dfrac{a_n x^n + \cdots}{b_n x^n + \cdots} \xrightarrow{x \to \pm\infty} \dfrac{a_n}{b_n} = \text{H.A.}$

$y = \dfrac{a_n}{b_n}$

H.A. for $\dfrac{5x^3 + 7x^2 - 1}{7x^3 - 4x^2 + 2x} \xrightarrow{|x| \to \infty} \boxed{\dfrac{5}{7} = y}$

Book Way $\dfrac{x^3\left(5 + \frac{7}{x} - \frac{1}{x^3}\right)}{x^3\left(7 - \frac{4}{x} + \frac{2}{x^2}\right)} \xrightarrow{x \to \pm\infty} \dfrac{5}{7}$

2. $\dfrac{a_{n+m} x^{n+m} + \cdots}{b_n x^n + \cdots}$ Do Long Division

This gives a slant or oblique asymptote

$$\frac{x^2 + 2x + 5}{x+1}$$

$$\begin{array}{r} x + 1 \quad r\,4 \\ x+1 \enclose{longdiv}{x^2 + 2x + 5} \\ -(x^2 + x) \\ \hline x + 5 \\ -(x + 1) \\ \hline 4 \end{array}$$

$$\begin{array}{r|rrr} -1 & 1 & 2 & 5 \\ & & -1 & -1 \\ \hline & 1 & 1 & 4 \\ & x & c & r \end{array}$$

This says $x^2 + 2x + 5 = (x+1)(x+1) + 4$

$$\frac{x^2 + 2x + 5}{x+1} = x + 1 + \frac{4}{x+1} \xrightarrow{|x| \to \infty} x + 1$$

$\boxed{\text{O.A.}: y = x + 1}$

$R(x) = \dfrac{x^2+2x+5}{x+1}$

$D: \mathbb{R} \setminus \{-1\}$

$\boxed{\text{V.A.} \quad x = -1}$

$\boxed{\text{O.A.} : \; y = x+1}$ (on prev. pg.)

x-int: $x^2+2x+5 = 0$

$x^2+2x = -5$

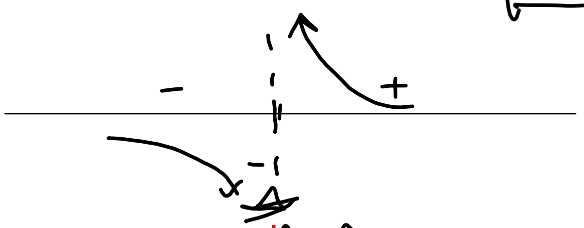
$x^2+2x+1^2 = -5+1 = -4$

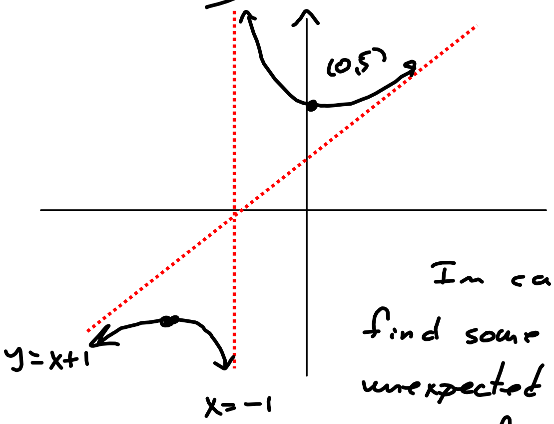
No real sol'n

$b^2-4ac = 2^2-4(1)(5) = 4-20 = -16$

No x-intercepts.

y-int: $R(0) = \frac{5}{1} = 5 \rightarrow \boxed{(0,5) = \text{y-int}}$





In calculus, we may find some of these have unexpected wiggle.
We can also find max/min points by taking $R'(x) \overset{\text{SET}}{=} 0$.

$$\frac{(x+5)(x-6)}{(x+3)(x-8)} = \frac{x^2-x-30}{x^2-5x-24} = R(x)$$

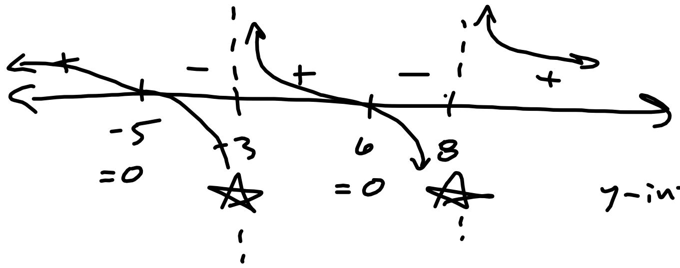
$D = \mathbb{R} \setminus \{-3, 8\}$

$-5, -3, 6, 8$

V.A.: $\boxed{x=-3,\ x=8}$

H.A.: $\frac{x^2}{x^2} \xrightarrow{x \to \pm\infty} \boxed{1 = y}$ H.A.

$\boxed{\text{x-int: } (-5,0),\ (6,0)}$



y-int: $\frac{-30}{-24} = \frac{5}{4}$

$(0, 5/4)$