Regard *x* as the independent variable and *y* as the dependent variable.

Find $\frac{dy}{dx}$ (y' is shorter)

Assume that $y=f(x)$, at least locally in at least a few (or a lot of) places

$$\underbrace{y\sec(x)}_{fg} = 5x\underbrace{\tan(y)}_{\text{Chain Rule!}}$$

$(fg)' = f'g + fg'$ = y is a function!

$$y'\sec(x) + y\sec(x)\tan(x) = 5\tan(y) + 5x\sec^2(y)y'$$

Let $f = \tan(y)$

$= f(u(x))$, where $y = u(x)$ ⟶

$$\frac{df}{du} = \sec^2(u)$$

$$\frac{du}{dx} = y' = \frac{dy}{dx}$$

$$\Rightarrow \frac{d}{dx}[\tan(y)] = \sec^2(y)\cdot y'$$

$$y\sec(x) = 5x\tan(y)$$

$$y'\sec(x) + y\sec(x)\tan(x) = 5\tan(x) + 5x\,\underline{\sec^2(y)y'}$$

Chain Rule on $\tan(y)$

$$\Rightarrow y'(\sec(x) - 5x\sec^2(y)) = 5\tan(x) - y\sec(x)\tan(x)$$

$$\Rightarrow \boxed{y' = \frac{5\tan(x) - y\sec(x)\tan(x)}{\sec(x) - 5x\sec^2(y)}}$$

25–32 Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

25. $y \sin 2x = x \cos 2y$, $(\pi/2, \pi/4)$

$(y = \frac{1}{2}x$

$y'\sin(2x) + y\,(\underbrace{2}_{\frac{du}{dx}}\underbrace{\cos(2x)}_{\cos(u)}) + \ldots$

$y'\sin(2x) + y\,(\cos(2x)) \cdot 2$

$\sin(2x) = f(u(x))$

$\Rightarrow u(x) = 2x \Rightarrow \frac{du}{dx} = 2$

$f(u) = \sin(u) \rightarrow$

$\frac{df}{du} = \cos(u)$

$y'\sin(2x) + y\,(2\cos(2x)) = 1 \cdot \cos(2y) + x\,(-\sin(2y) \cdot 2y'$

$\Rightarrow$

$y'\,(\sin(2x) + 2y\cos(2x) + 2x\sin(2y)) = -2y\cos(2x) + \cos(2y)$

$\Rightarrow y' = \dfrac{-2y\cos(2x) + \cos(2y)}{\sin(2x) + 2y\cos(2x) + 2x\sin(2y)}$

Now find an eq'n of the tangent line @ $(\frac{\pi}{2}, \frac{\pi}{4})$

$m_{tan} = y'(\frac{\pi}{2}) = y'\Big|_{x=\frac{\pi}{2}} = \dfrac{-2(\frac{\pi}{4})\cos(\pi) + \cos(\frac{\pi}{2})}{\sin(\pi) + 2(\frac{\pi}{4})\cos(\pi) + 2(\frac{\pi}{2})\sin(\frac{\pi}{2})}$

$= \dfrac{(-\frac{\pi}{2})(-1) + 0}{0 + \frac{\pi}{2}(-1) + \pi(-1)} = \dfrac{-\frac{\pi}{2}}{-\frac{\pi}{2} - \pi(\frac{2}{2})} = \dfrac{-\frac{\pi}{2}}{-\frac{3\pi}{2}} = \frac{\pi}{2} \cdot \frac{2}{3\pi} = \frac{1}{3}$

$\boxed{y = \frac{1}{3}(x - \frac{\pi}{2}) + \frac{\pi}{4}}$ STOP!

$= \frac{1}{3}x - \frac{\pi}{6} \cdot \frac{2}{2} + \frac{\pi}{4} \cdot \frac{3}{3} = \frac{1}{3}x + \frac{-2\pi + 3\pi}{12}$

$= \boxed{\frac{1}{3}x + \frac{\pi}{12}}$

Practice problems for implicit differentiation

Section 2.6

5–20 Find dy/dx by implicit differentiation.

5. $x^2 - 4xy + y^2 = 4$

7. $x^4 + x^2y^2 + y^3 = 5$

9. $\dfrac{x^2}{x+y} = y^2 + 1$

11. $y \cos x = x^2 + y^2$

13. $\sqrt{x+y} = x^4 + y^4$

15. $\tan(x/y) = x + y$

17. $\sqrt{xy} = 1 + x^2y$

19. $\sin(xy) = \cos(x + y)$

$x^2 - 4xy + y^2 = 4$

$\cos(xy)\,(1 + xy')$

5. $y' = \dfrac{2y - x}{y - 2x}$ **7.** $y' = -\dfrac{2x(2x^2 + y^2)}{y(2x^2 + 3y)}$

9. $y' = \dfrac{x(x + 2y)}{2x^2y + 4xy^2 + 2y^3 + x^2}$ **11.** $y' = \dfrac{2x + y \sin x}{\cos x - 2y}$

13. $y' = \dfrac{1 - 8x^3\sqrt{x+y}}{8y^3\sqrt{x+y} - 1}$ **15.** $y' = \dfrac{y \sec^2(x/y) - y^2}{y^2 + x \sec^2(x/y)}$

17. $y' = \dfrac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}$

19. $y' = -\dfrac{y\cos(xy) + \sin(x+y)}{x\cos(xy) + \sin(x+y)}$ **21.** $-\frac{16}{13}$

$x^2 - 4xy + y^2 = 4 \Rightarrow$

$2x - 4y - 4xy' + 2y \cdot y' = 0$

$(2y - 4x)y' = 4y - 2x$

$$\boxed{y' = \frac{4y-2x}{2y-4x}}$$

$\sqrt{x+y} = x^4 + y^4$

$(x+y)^{\frac{1}{2}}$

$\Rightarrow \frac{1}{2}(x+y)^{-\frac{1}{2}}(1+y') = 4x^3 + 4y^3y'$

$\Rightarrow \frac{1}{2}(x+y)^{-\frac{1}{2}} + \frac{1}{2}(x+y)^{-\frac{1}{2}}y' = 4x^3 + 4y^3y'$

$\Rightarrow \cancel{\frac{1}{2}y'\left(\frac{1}{\sqrt{x+y}}\right.}$

$\Rightarrow \left(\frac{1}{2}\underset{\frac{1}{\sqrt{x+y}}}{(x+y)^{-\frac{1}{2}}} - 4y^3\right)y' = 4x^3 + \cancel{4y^3y'} - \frac{1}{2}\underset{\frac{1}{\sqrt{x+y}}}{(x+y)^{-\frac{1}{2}}}$

$$\boxed{y' = \frac{4x^3 - \frac{1}{2}(x+y)^{-\frac{1}{2}}}{\frac{1}{2\sqrt{x+y}} - 4y^3}}$$

26. $\sin(x + y) = 2x - 2y, \quad (\pi, \pi)$

27. $x^2 - xy - y^2 = 1, \quad (2, 1)$ (hyperbola)

28. $x^2 + 2xy + 4y^2 = 12, \quad (2, 1)$ (ellipse)

29. $x^2 + y^2 = (2x^2 + 2y^2 - x)^2, \quad \left(0, \frac{1}{2}\right)$ (cardioid)

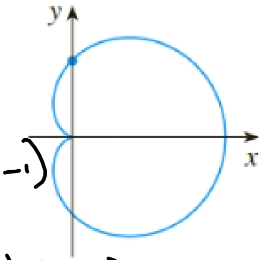
Cardiod is graphed in trig in polar coords

$x^2+y^2=r^2$

$x=r\cos\theta$

$y=r\sin\theta$

$r^2 = 2r^2\cos^2\theta + 2r^2\sin^2\theta - r\cos\theta$



#29

$2x + 2yy' = 2(2x^2+2y^2-x)'(4x+4yy'-1)$

$x + yy' = 2x^2\cdot 4x + 2x^2\cdot 4yy' + 2x^2(-1) + 2y^2\cdot 4x + 2y^2\cdot 4yy' + 2y^2(-1)$
$- x(4x) - x(4yy') - x(-1)$

$\Rightarrow \quad yy' = 8x^3 + 8x^2yy' - 2x^2 + 8xy^2 + 8y^3y' - 2y^2 - 4x^2 - 4xyy' + x$

$(y - 8x^2y - 8y^3 + 4xy)\,y' = 8x^3 - 2x^2 + 8xy^2 - 2y^2 - 4x^2 + x$

$$y' = \frac{8x^3 - 2x^2 + 8xy^2 + -2y^2 - 4x^2 + x}{y - 8x^2y - 8y^3 + 4xy}$$

$$y'\Big|_{\substack{x=0\\ y=\frac{1}{2}}} = y'\Big|_{(x,y)=(0,\frac{1}{2})} = \frac{0 - 0 + 0 - 2(\frac{1}{2})^2 - 0 + 0}{\frac{1}{2} - 0 - 8(\frac{1}{2})^3 + 0}$$

$$= \frac{\frac{1}{2}}{-1} = -\frac{1}{2} = m$$

$y = -\frac{1}{2}(x - 0) + \frac{1}{2}$

$= \frac{1}{2}x + \frac{1}{2}$

21. If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$.

Differentiate both Sides:

$$2x[f(x)]^3 + x^2\left(3[f(x)]^2 f'(x)\right) = 0$$

$f(1) = 2 \longrightarrow$

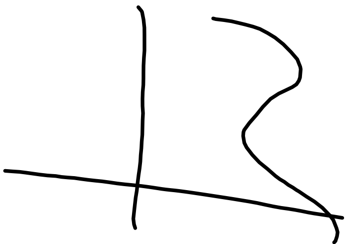
$$2(1)[2]^3 + 1^2\left(3(2^2)(f'(1)\right) = 0$$

$$16 + 12f'(1) = 0$$

$$f'(1) = -\frac{16}{12} = \boxed{-\frac{4}{3} = f'(1)}$$

Regard *y* as the independent variabe and *x* as the dependent variable. This is less standard, but worth looking at.

Find $\frac{dx}{dy}$ (x')



$x = f(y)$

$y\sec(x) = 5x\tan(y) \Longrightarrow$

$1\sec(x) + y\sec(x)\tan(x)\cdot x' = 5x'\tan(y) + 5x\sec^2(y)$

Solve for x' !

$(y\sec(x)\tan(x) - 5\tan(y))\,x' = 5x\sec^2(y) - \sec(x)$

$\Longrightarrow \quad x' = \dfrac{5x\sec^2(y) - \sec(x)}{y\sec(x)\tan(x) - 5\tan(y)}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Ohm's Law

Find $\frac{dR}{dt}$

$$R^{-1} = R_1^{-1} + R_2^{-1} \longrightarrow$$

$$\longrightarrow -R^{-2} \cdot R' = -R_1^{-2} \cdot R_1' - R_2^{-2} R_2' \longrightarrow$$

$$R' = \frac{-R_1^{-2} R_1' - R_2^{-2} R_2'}{-R^{-2}} = R^2 \left(R_1^{-2} R_1' + R_2^{-2} R_2' \right)$$

$$\frac{1}{R^{-2}} = R^2$$

$$R^2 \left(\frac{R_1'}{R_1^2} \cdot \frac{R_2^2}{R_2^2} + \frac{R_2'}{R_2^2} \cdot \frac{R_1^2}{R_1^2} \right)$$

$$= R^2 \left(\frac{R_1' R_2^2 + R_2' R_1^2}{R_1^2 R_2^2} \right)$$