$$\frac{d}{dx}[\text{constant}] = 0$$

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

$$\frac{d}{dx}[f \pm g] = \frac{df}{dx} \pm \frac{dg}{dx}$$

$$\frac{d}{dx}[(\text{constant})f(x)] = \text{constant}\,\frac{df}{dx}$$

$$\frac{d}{dx}[\sin(x)] = \cos(x)$$

$$\frac{d}{dx}[\cos(x)] = -\sin(x)$$

$$\frac{d}{dx}[\tan(x)] = \sec^2(x)$$

$$\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$$

$$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$$

$$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$$

$$\frac{d}{dx}[f(x)\,g(x)] = \frac{df}{dx}\cdot g(x) + f(x)\cdot\frac{dg}{dx}$$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Semi-important and key limits used...

$$\lim_{h\to 0} \frac{\sin h}{h} = 1$$

$$\lim_{h\to 0} \frac{\cos(h) - 1}{h} = 0 \ ?$$

§2.4 Question

#s1-3 Differentiate the following with respect to x

Find $\frac{dy}{dx}$.

Find $F'(x)$

① $y = x^2\cos(x)$

$f = x^2 \longrightarrow f' = 2x$

$g = \cos(x) \qquad g' = -\sin(x)$

$y' = (fg)' = f'g + fg' = \boxed{(2x)(\cos(x)) + (x^2)(-\sin(x)) = y'}$

② $y = \frac{x^2\sin(x)}{x+\cos(x)} = \frac{f}{g} = \frac{r\cdot s}{g}$

$f = r\cdot s \longrightarrow f' = r's + rs' = \boxed{(2x)(\sin(x)) + (x^2)(\cos(x)) = f'}$

$s = \sin(x) \longrightarrow s' = \cos(x)$

$r = x^2 \qquad r' = 2x$

$g = x+\cos(x) \longrightarrow g' = 1 + (-\sin(x))$

$y' = \frac{f'g - fg'}{g^2} = \frac{((2x)(\sin(x)) + (x^2)(\cos(x)))(x+\cos(x)) - (x^2\sin(x))(1-\sin(x))}{(x+\cos(x))^2}$

③ $y = f(x)g(x)h(x) = \underbrace{(f(x)g(x))}_{r}\underbrace{h(x)}_{s}$

$y' = r's + rs'$

$r = fg \Rightarrow r' = f'g + fg'$

$\Rightarrow y' = (f'g + fg')h(x) + (f(x)g(x))h'(x)$

$= f'gh + fg'h + fgh'$

$(fghj)' = f'ghj + fg'hj + fgh'j + fghj'$

$x^2\sec\theta\tan\theta = y \longrightarrow$

$y' = (2x)(\sec\theta\tan\theta) + x^2(\sec\theta\tan\theta)\tan\theta + x^2\sec\theta\sec^2\theta$

$\frac{d}{d\theta}[\sec\theta]$

Claim: $(\sec\theta)' = \sec\theta\tan\theta$

$$\frac{d}{d\theta}\left[\sec\theta\right] = \frac{d}{d\theta}\left[\frac{1}{\cos\theta}\right]$$

$f = 1 \qquad f' = 0$

$g = \cos\theta \qquad g' = -\sin\theta$

$$\Rightarrow \frac{d}{d\theta}\left[\sec\theta\right] = \frac{f'g - fg'}{g^2} = \frac{0 \cdot \cos\theta - 1(-\sin\theta)}{\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos^2\theta} = \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta} = \sec\theta\tan\theta. \ \blacksquare$$

$$f(x) = \frac{\omega N}{\sin(x) - \frac{1}{2}\cos(x)}$$ $\boxed{0 \le x < \frac{\pi}{2}}$ → Nah. No restriction.

Find where $f(x)$ has $\boxed{\text{a horizontal tangent}}$ $f'(x) = 0$

ω, N are assumed/given as constants.

$$\rightarrow f'(x) = \frac{(0)\cancel{(}\sin(x) - \frac{1}{2}\cos(x)) - \omega N\left(\cos(x) - \frac{1}{2}(-\sin(x))\right)}{\left(\sin(x) - \frac{1}{2}\cos(x)\right)^2} \overset{\text{SET}}{=} 0$$

$$\frac{A}{B} = 0 \iff A = 0$$

$$\rightarrow -\omega N\left(\cos(x) + \frac{1}{2}\sin(x)\right) = 0 \rightarrow$$

$$\cos(x) + \frac{1}{2}\sin(x) = 0 \rightarrow$$

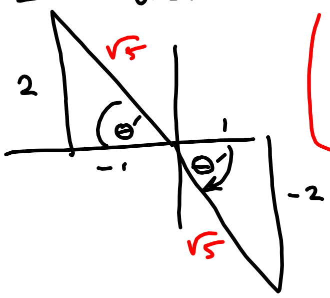
$$\cos(x) = -\frac{1}{2}\sin(x) \rightarrow$$

$$\frac{\cos(x)}{\sin(x)} = -\frac{1}{2} = \cot(x)$$

$3x - 6 = 0$

$x = 2$

$$\rightarrow \tan(x) = -2$$

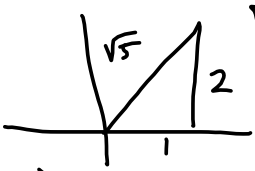


$\boxed{\text{Neither one is in } [0, \frac{\pi}{2})}$

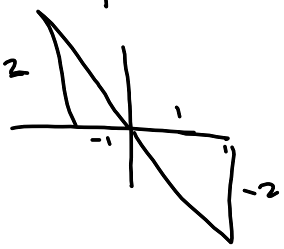
Question is poorly posed.

Find 2 solutions in $[0, 2\pi)$

$\arctan(-2) \notin [0, 2\pi)$



$\boxed{\arctan(2) = \theta'}$



$\boxed{x = \pi - \theta', \quad 2\pi - \theta'}$

θ' = reference angle in QI

Other ways:

$2\pi + \arctan(-2)$ $\qquad$ $\pi - \arctan(2)$

$\pi + \arctan(-2)$ $\qquad$ $2\pi - \arctan(2)$

$\boxed{\text{All sol'ns}}$

Dummy's way:

$\pi - \arctan(2) + 2n\pi = (2n+1)\pi - \arctan(2)$

$2\pi - \arctan(2) + 2n\pi = (2n+2)\pi - \arctan(2)$

OR $2(n+1)\pi - \arctan(2)$

More elegant: Sol'ns are π apart.

$\pi - \arctan(2) + n\pi = (n+1)\pi - \arctan(2), \; n \in \mathbb{Z}$

Solution as a set or collection

$\{ (2n+2)\pi - \arctan(2) \mid n \in \mathbb{Z} \}$

$\mathbb{Z} = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$ = integers

$\mathbb{N} = \{ 1, 2, 3, \ldots \}$

$\mathbb{Q} = \{ \frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0 \}$

$\mathbb{R} = \{ x \mid x \text{ is real} \} = (-\infty, \infty)$

§2.5 Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

$\frac{df}{dx} = \frac{df}{dx}\cdot\frac{dg}{dg} = \frac{df}{dg}\cdot\frac{dg}{dx}$ is how it works, but

be ware multiplying and dividing by dg or Δg.

$$h'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta h}{\Delta x}$$

$$(y = h(x))$$

$$\Delta y = f(x+\Delta x) - f(x)$$

$$\Delta x = x + \Delta x - \Delta x$$

Chain Rule.

$F(x) = f(g(x))$, where $g(x)$ is differentiable at a and

$f(g(x))$ is differentiable at $b = g(a)$. Then

$$\frac{dF}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx} \doteq \frac{df}{du} \cdot \frac{du}{dx}$$

Let $u = g(x)$

$$\Delta u = g(x+\Delta x) - g(x)$$

$$\frac{\Delta u}{\Delta x} = \frac{g(x+\Delta x) - g(x)}{\Delta x} = g'(x) + \varepsilon_1$$

This makes sense. $g(x)$ is difb$^{\underline{l}}$

$$\frac{\Delta u}{\Delta x} \xrightarrow{\Delta x \to 0} g'(x)$$

$\Delta u = g'(x) \Delta x + \varepsilon_1 \Delta x$ NOTE $\varepsilon_1 \to 0$ as $\Delta x \to 0$

Define $\varepsilon_1 = 0$ at $\Delta x = 0$.

This makes ε_1 continuous.

$y = f(u)$

$$\Delta y = f(u+h) - f(u)$$

$\vdots$

$$\frac{\Delta y}{\Delta u} = f'(u) + \varepsilon_2$$

$$\Delta y = (f'(u) + \varepsilon_2) \Delta u$$

$$= (f'(u) + \varepsilon_2)(g'(a) + \varepsilon_1) \Delta x$$

$$\frac{\Delta y}{\Delta x} = (f'(u) + \varepsilon_2)(g'(a) + \varepsilon_1)$$

$\varepsilon_1, \varepsilon_2 \longrightarrow 0$ as $\Delta x \to 0$

$$\xrightarrow{\Delta x \to 0} f'(u) g'(a)$$