Power Rule

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

$10x^{\frac{5}{3}} - 11x^4 = f(x) \Longrightarrow$

$f'(x) = \left(\frac{5}{3}\right)(10)\, x^{\frac{2}{3}} - 44x^3$

$f'(x) = \frac{df}{dx}$ is Liebniz Notation

$\frac{d}{dx}\left[10x^{\frac{5}{3}} - 11x^4\right] = \frac{50}{3}x^{\frac{2}{3}} - 44x^3$

It indicates the variable with respect to which you're differentiating

$\frac{d}{dz}\left[x^5\right] = 0$ No "z" in $f(x)$.

$f(☺) = ☺^5 - 7☺^2$

$\frac{df}{d\theta} = 5☺^4 - 14☺$

$f(3x-7) = 10(3x-7)^4 + 11(3x-7)^3 - 4(3x-7)$

$\frac{df}{d(3x-7)} = 40(3x-7)^3 + 33(3x-7)^2 - 4(3x-7)$

If ~~f(x)~~ = f(u(x)) is a function of a function of x,

then $\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$f(x) = 40(3x-7)^3 + 33(3x-7)^2 - 4(3x-7)$

$\Rightarrow \frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$, where $u(x) = 3x-7$

$= \left((120)(3x-7)^2 + 66(3x-7)^1 + 4\right)(3)$

$f(x) = \sqrt{6-x} = (6-x)^{\frac{1}{2}}$

$u(x) = 6-x$

$f(u) = \sqrt{u} = u^{\frac{1}{2}}$

$\frac{df}{dx}\left[\left(\frac{1}{2}u^{-\frac{1}{2}}\right)(-1)\right] = -\frac{1}{2}(6-x)^{-\frac{1}{2}} = -\frac{1}{2\sqrt{6-x}}$

$= -\frac{1}{2}\left(\frac{1}{(6-x)^{1/2}}\right) =$

$$f(x) = (x^2 - 5x + 2)^{53}$$

$$\Rightarrow \underbrace{53(x^2 - 5x + 2)^{52}}_{\frac{df}{du}} \underbrace{(2x - 5)}_{\frac{du}{dx}} = f'(x),$$

where $u(x) = x^2 - 5x + 2$

$G(t) = \dfrac{1 - 4t}{5 + t}$ Find $G'(t)$

$$\frac{G(c) - G(t)}{c - t} = \frac{1}{c-t}\left[\frac{1-4c}{5+c} - \frac{1-4t}{5+t}\right]$$

$$= \frac{1}{c-t}\left[\frac{1-4c}{c+5} \cdot \frac{t+5}{t+5} - \frac{1-4t}{t+5} \cdot \frac{c+5}{c+5}\right]$$

$$= \frac{1}{c-t}\left[\frac{t+5-4ct-20c - (c+5-4ct-20t)}{(t+5)(c+5)}\right]$$

$$= \frac{1}{c-t}\left[\frac{t+\cancel{5}-4ct-20c-c-\cancel{5}+4ct+20t}{(\quad)}\right]$$

$$= \frac{1}{c-t}\left[\frac{t-21c+20t}{(\quad)}\right] = \frac{1}{c-t}\left[\frac{21t-21c}{(\quad)}\right]$$

$$= \frac{1}{c-t}\left[\frac{21(t-c)}{(c+5)(t+5)}\right] = \frac{1}{c-t}\left[\frac{-21(c-t)}{(c+5)(t+5)}\right] = \frac{-21}{(c+5)(t+5)}$$

$$\xrightarrow{c \to t} \frac{-21}{(t+5)(t+5)} = \boxed{\frac{-21}{(t+5)^2} = G'(t)}$$

Product Rule

$(fg)' = f'g + fg'$

$(x^2-3)(x-7) = f(x)g(x) = h(x) \longrightarrow$

$h'(x) = f'g + fg' = (2x)(x-7) + (x^2-3)(1)$

$H(t) = \frac{4t-1}{t+5} = (4t-1)(t+5)^{-1} = f(t)g(t) \longrightarrow$

$H'(t) = f'g + fg' = (4)(t+5)^{-1} + (4t-1)\boxed{(-1(t+5)^{-2}(1))}$

$= \frac{4}{t+5} - \frac{4t-1}{(t+5)^2}$ $\quad -1u^{-2} \cdot \frac{du}{dt}$

$H(t) = -G(t)$
from previous example

$= \frac{4}{t+5} \cdot \frac{t+5}{t+5} - \frac{4t-1}{(t+5)^2}$

$= \frac{4t+20-4t+1}{(t+5)^2} = \frac{21}{(t+5)^2}$

$\frac{d}{dt}\left[(t+1)^{-1}\right] =$

$\frac{df}{du} \cdot \frac{du}{dt}$

$f(u) = u^{-1} \longrightarrow \frac{df}{du} = -1u^{-2}$

$u(t) = t+1 \longrightarrow \frac{du}{dt} = 1$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

"Proof"

$$\left(\frac{f}{g}\right)' = \left(f\, g^{-1}\right)' = (fh)' = f'h + fh'$$

$$h = g^{-1}$$

$$f' = f'$$

$$h' = \frac{d}{dx}\left[\, g^{-1}\right] = \left(-1 g^{-2}\right)\left(g'\right) = -\frac{g'}{g^2}$$

So $$\left(\frac{f}{g}\right)' = f'h + fh' = f' \cdot g^{-1} + f\left(-\frac{g'}{g^2}\right)$$

$$= \frac{f'}{g} \cdot \frac{g}{g} - \frac{fg'}{g^2} = \frac{f'g - fg'}{g^2}$$

$$h(t) = \frac{4t-1}{t+5} =$$

$$f = 4t-1 \qquad f' = 4$$

$$g = t+1 \qquad g' = 1$$

$$\Rightarrow h'(t) = \frac{4(t+5) - (4t-1)(1)}{(t+5)^2} = \frac{4t+20-4t+1}{(t+5)^2} = \frac{21}{(t+5)^2}$$

§2.2

$f(x) = x^{\frac{1}{3}} \implies f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}$

$= \frac{1}{3(\sqrt[3]{x})^2}$, depends on domain, sometimes.

$\boxed{\frac{c^{\frac{1}{3}} - x^{\frac{1}{3}}}{c - x} = f(x)}$ $\quad (x^{\frac{1}{3}})^2 = x^{(\frac{1}{3})(2)} = x^{\frac{2}{3}}$

$(a-b)(a^2+ab+b^2) = a^3 - b^3$

$(a+b)(a^2-ab+b^2) = a^3+b^3$

2 ways:

$$\frac{(c^{\frac{1}{3}} - x^{\frac{1}{3}})(c^{\frac{2}{3}} + c^{\frac{1}{3}}x^{\frac{1}{3}} + x^{\frac{2}{3}})}{(c-x)(c^{\frac{2}{3}} + c^{\frac{1}{3}}x^{\frac{1}{3}} + x^{2/3})} = \frac{(c^{\frac{1}{3}})^3 - (x^{\frac{1}{3}})^3}{(c-x)(c^{2/3} + c^{1/3}x^{1/3} + x^{2/3})} = \frac{\cancel{c-x}}{\cancel{(c-x)}(c^{2/3} + c^{1/3}x^{1/3} + x^{2/3})}$$

$$\xrightarrow{c \to x} \frac{1}{x^{2/3} + x^{2/3} + x^{2/3}} = \boxed{\frac{1}{3x^{2/3}}}$$

METHOD 2

$$\frac{c^{\frac{1}{3}} - x^{\frac{1}{3}}}{c - x} = \frac{c^{\frac{1}{3}} - x^{\frac{1}{3}}}{(c^{\frac{1}{3}})^3 - (x^{\frac{1}{3}})^3} = \frac{\cancel{c^{\frac{1}{3}} - x^{\frac{1}{3}}}}{\cancel{(c^{\frac{1}{3}} - x^{\frac{1}{3}})}(c^{\frac{2}{3}} + c^{\frac{1}{3}}x^{\frac{1}{3}} + x^{2/3})}$$

$$\xrightarrow{c \to x} \frac{1}{x^{2/3} + x^{2/3} + x^{2/3}} = \boxed{\frac{1}{3x^{2/3}} = f'(x)}$$

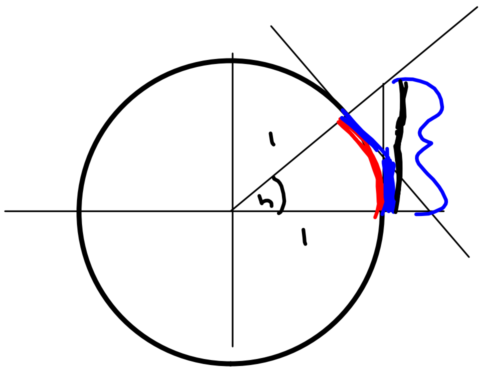
§2.2 #14

Remember $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$$f(x) = x|x| = \begin{cases} x \cdot x = x^2 & \text{if } x \geq 0 \\ x \cdot (-x) = -x^2 & \text{if } x < 0 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x^2)$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x^2)$$



h = angle = arc length

<

$\frac{d}{dx}[\sin(x)] = \cos(x)$

$h < \tan h$

Monday or Tuesday

$\frac{d}{dx}[\cos(x)] = -\sin(x)$