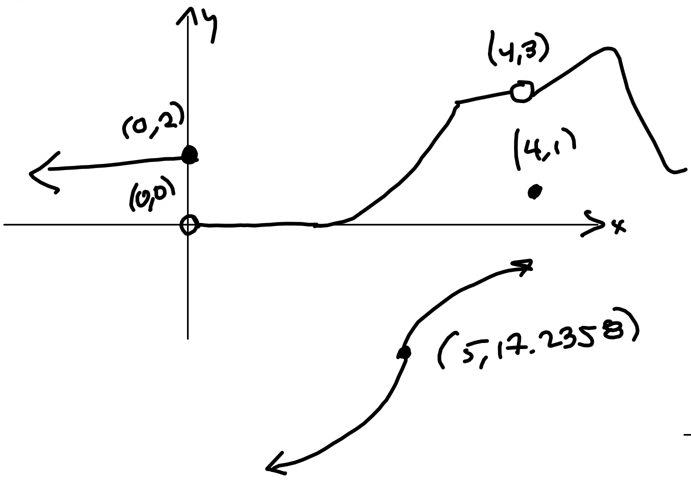
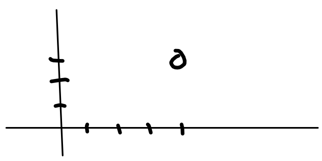
15–18 Sketch the graph of an example of a function f that satisfies all of the given conditions.

18. $\lim_{x\to 0^-} f(x) = 2,\quad \lim_{x\to 0^+} f(x) = 0,\quad \lim_{x\to 4^-} f(x) = 3,$
$\lim_{x\to 4^+} f(x) = 0,\quad f(0) = 2,\quad f(4) = 1$





16. $\lim_{x\to 0} f(x) = 1,\quad \lim_{x\to 3^-} f(x) = -2,\quad \lim_{x\to 3^+} f(x) = 2,$
$f(0) = -1,\quad f(3) = 1$

Section 1.7 #s 19 and 20 are, I would say, beyond the scope. Something that would take most of a lecture to explain and for people to absorb. They're the kinds of problems you'd see in Advanced Calculus, if at all.

1.7 #20

$\lim_{x\to a} \sqrt{x} = \sqrt{a}$ if $a > 0$

$f(x) = \sqrt{x}$

$L = \sqrt{a}$

Want $|f(x) - L| < \varepsilon$

iff $|\sqrt{x} - \sqrt{a}| < \varepsilon$

iff $|\sqrt{x} - \sqrt{a}| \cdot \left|\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}\right| = \frac{|\sqrt{x}^2 - \sqrt{a}^2|}{\sqrt{x}+\sqrt{a}} < \varepsilon$

iff $\frac{|x-a|}{\sqrt{x}+\sqrt{a}} < \varepsilon$

Assume $|x-a| < \frac{1}{2}a$

i.e. $-\frac{1}{2}a < x - a < \frac{1}{2}a$

i.e. $\frac{1}{2}a < x < \frac{3}{2}a$

iff $\sqrt{\frac{1}{2}a} < \sqrt{x} < \sqrt{\frac{3}{2}a}$

$\Rightarrow \sqrt{x} + \sqrt{a} > \sqrt{\frac{1}{2}a} + \sqrt{a}$

So $\frac{|x-a|}{\sqrt{x}+\sqrt{a}} < \frac{|x-a|}{\sqrt{\frac{1}{2}a}+\sqrt{a}} < \varepsilon$ iff

$|x-a| < \frac{\varepsilon}{\sqrt{\frac{1}{2}a}+\sqrt{a}}$

$\lim_{x \to c} f(x)$ & $\lim_{x \to c} g(x)$ $\nexists$, but

$\lim_{x \to c} \left(f(x) + g(x)\right)$ $\exists$

$f(x) = \sin\left(\frac{1}{x}\right)$, $g(x) = -\sin\left(\frac{1}{x}\right)$ $\longrightarrow$

$\lim_{x \to 0} \left(f(x) + g(x)\right) = \lim_{x \to 0} 0 = 0.$

No uploading of §1.6 #30.