You need the circumference of a metal disk you're making to be 25 cm, within +/- 0.01 cm.

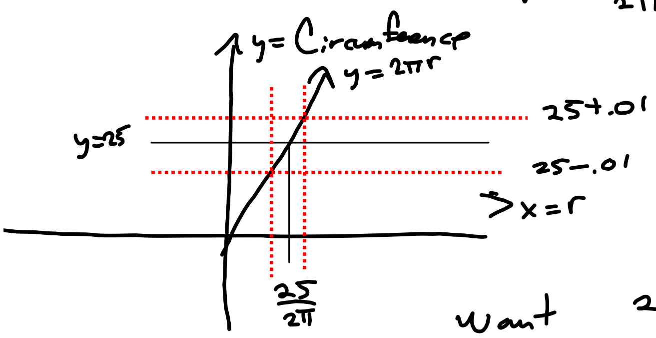
The only adjustment you have on the machine is the radius. How close do you have to come to a the exact radius needed in order to keep the circumference within +/- 0.01 cm of 25 cm?

$2\pi r$ = circumference

Let r = radius of the disk (in cm)

We want $2\pi r = 25$ cm, i.e.

$$r = \frac{25}{2\pi} \text{ cm}$$



within

want $25 - .01 < y < 25 + .01$

$24.99 < 2\pi r < 25.01$

Want $|2\pi r - 25| < .01$

what's that say about $\left|r - \frac{25}{25}\right|$?

Here's what it says:

$$2\pi \left|r - \frac{25}{2\pi}\right| < .01$$

$$\left|r - \frac{25}{2\pi}\right| < \frac{.01}{2\pi} \text{ of } r = \frac{25}{2\pi}$$

$2\pi \to$ slope of $y = 2\pi r$!

$\delta = \frac{.01}{2\pi}$ is our tolerance in the radius.

= radius of horizontal tube.

$r \in \left(\frac{25}{2\pi} - \delta, \frac{25}{2\pi} + \delta\right)$ will keep

us inside $C = (\underbrace{25 - .01, 25 + .01}_{\text{The epsilon tube.}})$ tube.

Want $|c - 25| < .01$ $\quad$ c = circumference

means

$c - 25 < .01$ and $c - 25 > -1$

$-.01 < c - 25 < .01$

$24.99 < c < 25.01$

$c = 2\pi r \longrightarrow$

$|2\pi r - 25| < .01$

$2\pi \left| r - \frac{25}{2\pi} \right| < .01$

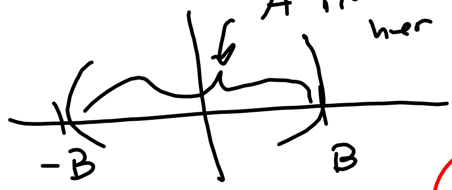
$\left| r - \frac{25}{2\pi} \right| < \frac{.01}{2\pi} = \frac{\varepsilon}{2\pi}$

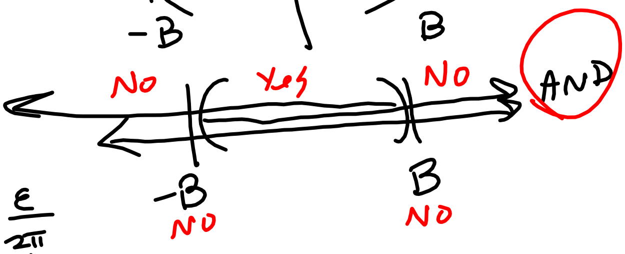
$= \frac{\text{Vertical Tolerance}}{\text{Growth rate}}$

$|A| < B \longrightarrow$

$A < B$ AND $A > -B$

A lives in here





$= (-B, B)$

$A \in (-B, B)$.

What about a tolerance of $\pm .001$?

$\left| r - \frac{25}{2\pi} \right| < \frac{.001}{2\pi}$

$\pm .0001$?

$\left| r - \frac{25}{2\pi} \right| < \frac{.0001}{2\pi}$

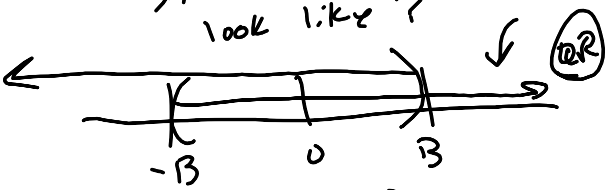
$\pm \varepsilon$?

$\left| r - \frac{25}{2\pi} \right| < \frac{\varepsilon}{2\pi} = \frac{\varepsilon}{\text{Growth rate}}$

$\left| r - \frac{25}{2\pi} \right| < \delta = \frac{\varepsilon}{2\pi} \longrightarrow$

$|2\pi r - 25| < \varepsilon$

What does $A < B$ OR $A > -B$ look like?

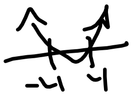


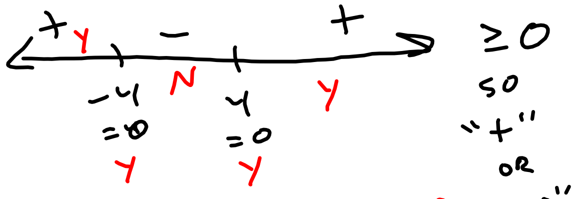
$= (-\infty, \infty)$

Homework:

$x^2 - 16 \geq 0$

$(x-4)(x+4)$





$= (-\infty, -4] \cup [4, \infty)$

$|A| \leq B \longrightarrow$

$A \leq B$ And $A \geq -B$

$-B \leq A \leq B$ is OK

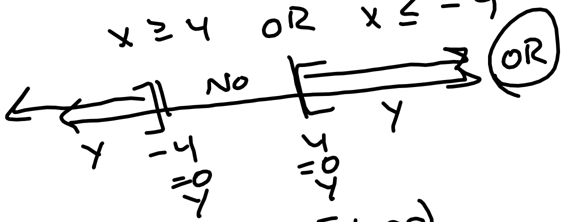
$x^2 - 16 \geq 0$

$x^2 \geq 16$

$\sqrt{x^2} \geq \sqrt{16}$

$|x| \geq 4 \longrightarrow$

$x \geq 4$ OR $x \leq -4$



$= (-\infty, -4] \cup [4, \infty)$

$|A| > B \longrightarrow$

$A > B$ OR $A < -B$

~~$-B > A > B$ is BAD!~~

~~$-7 > x > 7$~~

FACT: $\lim_{x\to 3}(5x-4) = 11$

How close does x have to be to 3 to keep $5x-4$

$\pm .1$ of 11? $\frac{.1}{5}$ $5 = $ slope!

$\pm .01$ of 11? $\frac{.01}{5}$

$\pm .001$ of 11? $\frac{.001}{5}$

$\pm \varepsilon$ of 11? $\delta = \frac{\varepsilon}{5}$

$\delta =$ how close to 3 x is
$|x-3| < \delta =$ upper bound on $\underbrace{\text{distance from } x \text{ to } 3}_{|x-3|}$.

Prove $\lim_{x\to 3}(5x-4) = 11$

Let $\varepsilon > 0$. Define $\delta = \frac{\varepsilon}{5}$. Then $0 < |x-3| < \delta$ implies

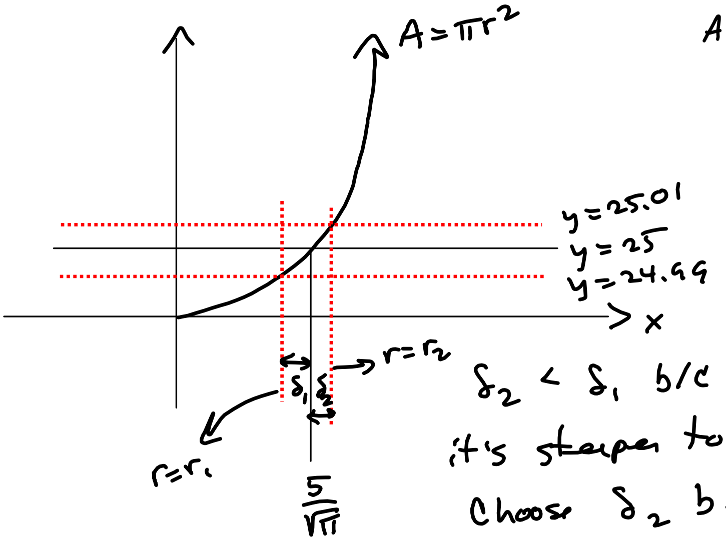
$|5x-4-11| = |5x-15| = 5|x-3| < 5\delta = 5\cdot\frac{\varepsilon}{5}$

$= \varepsilon$ ∎

$\boxed{D}$ $\lim_{x\to c} f(x) = L$ means given any $\varepsilon > 0$, there exists a $\delta > 0$ such that $0 < |x-c| < \delta$ implies $|f(x)-L| < \varepsilon$.

Pf Let $\varepsilon > 0$. Define $\delta = \frac{\varepsilon}{5}$. Then $0 < |x-3| < \delta \Rightarrow$
$|f(x)-L| = |5x-4-11| = |5x-15| = 5|x-3| < 5\delta = 5\left(\frac{\varepsilon}{5}\right) = \varepsilon$ ∎

What if you need the disk to be 25 cm^2 +/- 0.01 cm^2? This is trickier.



Area $= f(r) = \pi r^2$

$L = 25 \text{ cm}^2$

$\pi r^2 = 25 \Rightarrow$

$r^2 = \frac{25}{\pi} \Rightarrow$

$r = \pm \frac{5}{\sqrt{\pi}}$

$\delta_2 < \delta_1$ b/c it's steeper to the right

Choose δ_2 b/c it's smaller

Find δ_1 & δ_2:

$$|f(r) - L| = |\pi r^2 - 25| = .01$$

$$\pi \left|r^2 - \frac{25}{\pi}\right| = .01$$

$$\left|r^2 - \frac{25}{\pi}\right| = \frac{.01}{\pi}$$

$$r^2 - \frac{25}{\pi} = \pm \frac{.01}{\pi}$$

$$r^2 = \frac{25}{\pi} \pm \frac{.01}{\pi}$$

$$r = \pm\sqrt{\frac{25}{\pi} \pm \frac{.01}{\pi}} = \sqrt{\frac{25}{\pi} \pm \frac{.01}{\pi}}, \text{ since } r > 0$$

$$r_1 = \sqrt{\frac{25}{\pi} - \frac{.01}{\pi}}$$

$$r_2 = \sqrt{\frac{25}{\pi} + \frac{.01}{\pi}}$$

$$\delta = \min\left\{\left|\frac{5}{\sqrt{\pi}} - r_1\right|, \left|\frac{5}{\sqrt{\pi}} - r_2\right|\right\}$$

$$= \min\left\{\frac{5}{\sqrt{\pi}} - r_1, r_2 - \frac{5}{\sqrt{\pi}}\right\}$$

Short & Simple:

Find r_1 & r_2

Solve $\pi r^2 = 25.01$

$r^2 = \frac{25.01}{\pi} \longrightarrow r = \sqrt{\frac{25.01}{\pi}} \approx 2.82151205091$

$\pi r^2 = 24.99 \longrightarrow r = \sqrt{\frac{24.99}{\pi}} \approx 2.82038367172$

$\delta_1 = \left| 2.8203\ldots - \frac{5}{\sqrt{\pi}} \right|$ -0.000564246013793

$\delta_2 = \left| 2.8215\ldots - \frac{5}{\sqrt{\pi}} \right|$ 0.00056413317587

∴ δ_2 will make sure you're w/in the tolerance.