$$\lim_{x\to 2} \frac{x^2-3x+2}{x^2+3x-10} = \lim_{x\to 2} \frac{2^2-3(2)+2}{2^2+3(2)-10} = \frac{0}{0} \; ?!$$

If f is a polynomial, then $\lim_{x\to c} f(x) = f(c)$

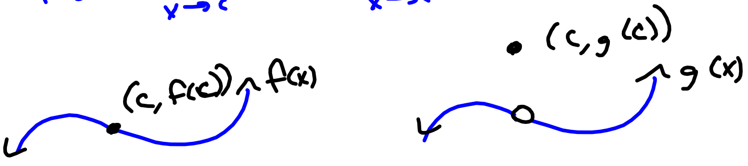
$$\frac{x^2-3x+2}{x^2+3x-10} = \frac{(x-1)(x-2)}{(x+5)(x-2)} = \underbrace{\frac{x-1}{x+5}}_{x\neq 2} \xrightarrow{x\to 2} \frac{2-1}{2+5} = \boxed{\frac{1}{7} = \lim_{x\to 2} f(x)}$$

$$\lim_{x\to 2} \frac{x^2-3x+2}{x^2+3x-10} = \lim_{x\to 2} \frac{(x-1)(x-2)}{(x+5)(x-2)}$$

$$= \lim_{x\to 2} \frac{(x-1)}{x+5} = \frac{2-1}{2+5} = \frac{1}{7}$$

If $f(x) = g(x)$, except possibly at $x = c$,

then $\lim_{x\to c} f(x) = \lim_{x\to c} g(x)$



This is why $\lim_{x\to 2} \frac{x^2-3x+2}{x^2+3x\underbrace{-2}_{-10}} = \lim_{x\to 2} \frac{x-1}{x+5}$

You can't say

$$\frac{x^2-3x+2}{x^2+3x-10} = \frac{x-1}{x+5}, \text{ w/o saying " } x \neq 2 \text{ "}$$

That's the only fly in the ointment, if you want to use my "pro-grade" limit steps, and eliminate a lot of extra writing of the limit symbol at every step. If you write limit, you must continue to write the limit symbol.

With my method, you don't write limit over and over; rather, you "pass to the limit" in the last step, making note of the fact that the fractions aren't EXACTLY equal at the limiting input, in this case, $x = 2$.

#14 $\lim_{\theta \to 0} \frac{\sin(3\theta)}{\tan(4\theta)}$

Book Way: **For now (1.5ish and some 1.6ish) we plug in values closer and closer to 0.**

$f(\theta) = \frac{\sin(3\theta)}{\tan(4\theta)}$

$f(.000001)$

$= 0.749999999995$

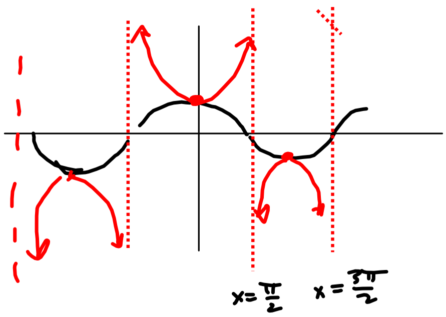
$f(-.000001)$

$= 0.749999999995$

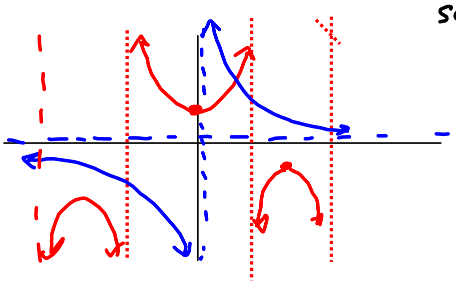
Conjecture: The limit is 3/4

Never analyzed this one. The idea in 1.5 was to just plug in numbers in Radians Mode on your calculator.

#16 $\lim_{x \to \frac{\pi}{2}^+} \frac{7}{x} \sec(x)$

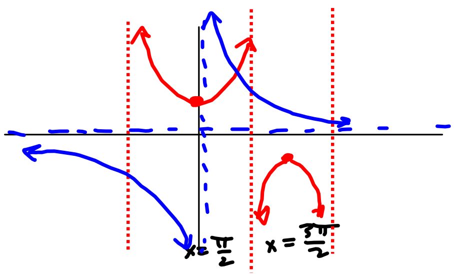


Graph of sec(x) using cos(x)



superimpose $y = \frac{7}{x}$

$\frac{7}{x} \sec(x)$



23. $\lim_{x \to 3} \dfrac{\dfrac{1}{x} - \dfrac{1}{3}}{x - 3}$

24. $\lim_{h \to 0} \dfrac{(3 + h)^{-1} - 3^{-1}}{h}$

25. $\lim_{t \to 0} \dfrac{\sqrt{1 + t} - \sqrt{1 - t}}{t}$

26. $\lim_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t} \right)$

27. $\lim_{x \to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$

28. $\lim_{x \to 2} \dfrac{x^2 - 4x + 4}{x^4 - 3x^2 - 4}$

29. $\lim_{t \to 0} \left(\dfrac{1}{t\sqrt{1 + t}} - \dfrac{1}{t} \right)$

30. $\lim_{x \to -4} \dfrac{\sqrt{x^2 + 9} - 5}{x + 4}$

31. $\lim_{h \to 0} \dfrac{(x + h)^3 - x^3}{h}$

32. $\lim_{h \to 0} \dfrac{\dfrac{1}{(x + h)^2} - \dfrac{1}{x^2}}{h}$

33. (a) Estimate the value of

$$\lim_{x \to 0} \frac{x}{\sqrt{1 + 3x} - 1}$$

by graphing the function $f(x) = x/(\sqrt{1 + 3x} - 1)$.

(b) Make a table of values of $f(x)$ for x close to 0 and guess the value of the limit.

(c) Use the Limit Laws to prove that your guess is correct.

$f(x) = \frac{1}{x^2} \Longrightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

32. $\lim_{h \to 0} \dfrac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h}$:

$(x+h)^2 = x^2 + 2xh + h^2$

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$\frac{1}{h}\left[\frac{1}{(x+h)^2} \cdot \frac{x^2}{x^2} - \frac{1}{x^2} \cdot \frac{(x+h)^2}{(x+h)^2}\right]$

$= \frac{1}{h}\left[\frac{x^2 - (x+h)^2}{x^2(x+h)^2}\right] = \frac{1}{h}\left[\frac{x^2 - (x^2 + \overset{+2xh}{xh} + h^2)}{x^2(x+h)^2}\right]$

$= \frac{1}{h}\left\{\frac{x^2 - x^2 \overset{-2xh}{- xh} \cdot h^2}{x^2(x+h)^2}\right] = \frac{1}{h}\left[\frac{-\overset{-2xh}{xh} - h^2}{x^2(x+h)^2}\right] = \frac{1}{h}\left(\frac{h[-2x-h]}{x^2(x+h)^2}\right)$

$= \frac{-2x-h}{x^2(x+h)^2} \xrightarrow{h \to 0} \frac{-2x}{x^2 \cdot x^2} = \frac{-2x}{x^4} = -\frac{2}{x^3}$

if $h \neq 0$

Good job, Andrew

33. (a) Estimate the value of

$$\lim_{x \to 0} \frac{x}{\sqrt{1 + 3x} - 1} = \lim_{x \to 0} f(x)$$

by graphing the function $f(x) = x/(\sqrt{1 + 3x} - 1)$.

(b) Make a table of values of $f(x)$ for x close to 0 and guess the value of the limit.

(c) Use the Limit Laws to prove that your guess is correct.

c involves real skill. The rest is just technology

$$\frac{x}{\sqrt{3x+1} - 1} = \frac{x}{\sqrt{3x+1} - 1} \cdot \frac{\sqrt{3x+1} + 1}{\sqrt{3x+1} + 1} = \frac{x(\sqrt{3x+1} + 1)}{3x+1-1}$$

$$(a-b)(a+b) = a^2 - b^2$$

$$= \frac{x(\sqrt{3x+1} + 1)}{3x} = \frac{\sqrt{3x+1} + 1}{3} \quad x \neq 0 \quad \xrightarrow{x \to 0} \frac{\sqrt{1} + 1}{3} \boxed{= \frac{2}{3} = \lim_{x \to 0} f(x)}$$

$\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = M$

$\lim(f\pm g) = \lim f \pm \lim g = L \pm M$

$\lim(fg) = \lim f \lim g$

$\lim\left(\frac{f}{g}\right) = \frac{\lim f}{\lim g} = \frac{L}{M}$, if $\lim g \neq 0$

If $f(x) = g(x)$, except @ $x = c$, then
$\lim f = \lim g$

If $f(x) < g(x)$, except @ $x = c$, then
$\lim f \leq \lim g$ *

If $\lim h = \lim f$ &
$f < g < h$ $\forall x$, except possibly $x = c$, SQUEEZE THEOREM
then $\lim g = \lim f = \lim h$

* on $[0,1)$ $f(x) = x^2 < x = g(x)$
& $\lim_{x\to 1} f(x) = \lim_{x\to 1} g(x)$

$\cos\theta < \frac{\sin\theta}{\theta} < 1$ on $(0, \frac{\pi}{2})$

$\lim_{\theta\to 0} \cos\theta = 1 \leq \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \leq \lim_{\theta\to 0} 1 = 1$

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ by Squeeze Theorem.

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§2.4 $\frac{d}{d\theta}[\sin\theta] = \cos\theta$!

$$\frac{\sin(\theta+h) - \sin\theta}{h} = \frac{\sin\theta\cos h + \sin h\cos\theta - \sin\theta}{h}$$

$$= \frac{\sin\theta(\cos h - 1) + \sin h\cos\theta}{h}$$

$$= \sin\underbrace{\boxed{\left(\frac{\cos h - 1}{h}\right)}}_{\to 0 \text{ as } h\to 0} + \frac{\sin h}{h}\cos\theta \xrightarrow{h\to 0} \cos\theta$$

You need the circumference of a metal disk you're making to be 25 cm, within +/- 0.01 cm.

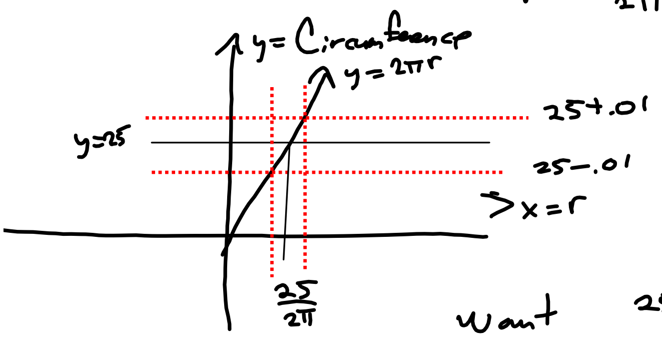
The only adjustment you have on the machine is the radius. How close do you have to come to a the exact radius needed in order to keep the circumference within +/- 0.01 cm of 25 cm?

$2\pi r$ = circumference

Let r = radius of the disk (in cm)

We want $2\pi r = 25$ cm, i.e.

$$r = \frac{25}{2\pi} \text{ cm}$$



within

want $25 - .01 < y < 25 + .01$

$24.99 < 2\pi r < 25.01$

Want $|2\pi r - 25| < .01$

what's that say about $|r - \frac{25}{25}|$?

Here's what it says:

$2\pi \left| r - \frac{25}{2\pi} \right| < .01$

$\left| r - \frac{25}{2\pi} \right| < \frac{.01}{2\pi}$ of $r = \frac{25}{2\pi}$

slope of $y = 2\pi r$!

$\delta = \frac{.01}{2\pi}$ is our tolerance in the radius.

= radius of horizontal tube.

$r \in \left(\frac{25}{2\pi} - \delta, \frac{25}{2\pi} + \delta\right)$ will keep us inside $C = \underbrace{(25 - .01, 25 + .01)}_{\text{The epsilon tube.}}$ tube.

FACT: $\lim_{x \to 3} (5x-4) = 11$

How close does x have to be to 3 to keep $5x-4$

$\pm .1$ of 11 ?

$\pm .01$ of 11 ?

$\pm .001$ of 11?

$\pm \varepsilon$ of 11?

What if you need the disk to be 25 cm^2 +/- 0.01 cm^2? This is trickier.