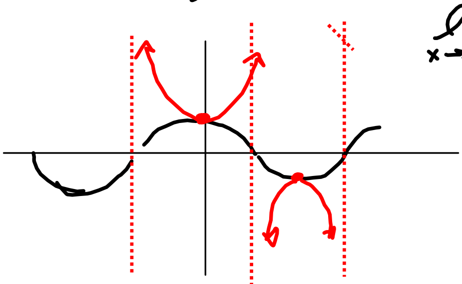
§1.5

https://www.desmos.com/calculator

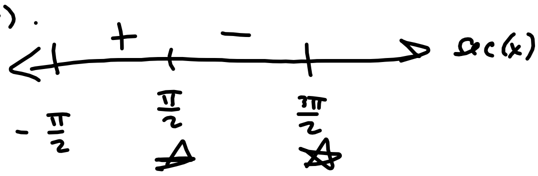
15 $\lim_{x\to 4^+} \frac{x+5}{x-4}$

#14 $\lim_{\theta\to 0} \frac{\sin(3\theta)}{\tan(4\theta)}$

#16 $\lim_{x\to \frac{\pi}{2}^+} \frac{7}{x}\sec(x)$



$\lim_{x\to\frac{\pi}{2}^+} \sec(x)$.



$\lim_{x\to\frac{\pi}{2}^+} \sec(x) = +\infty$

$\lim_{x\to\frac{\pi}{2}^+} \frac{1}{x}\sec(x)$ is $\frac{-\infty}{\frac{\pi}{2}}$ situation

$= -\infty$ sitch.

$\frac{5}{x^3-1}$

To analyze $f(x) = \frac{1}{x^3-27}$, we need to understand the behavior of x^3-1

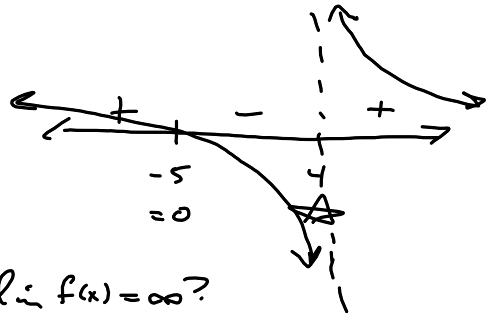
$\lim_{x\to 3^+} f(x)$, $\lim_{x\to 3^-} f(x)$.

$x^3-y^3 = (x-y)\underbrace{(x^2+xy+y^2)}_{\text{No real zeros}} \overset{\text{SET}}{=} 0 \Rightarrow x = y$

§1.5

\# 15 $\lim_{x\to 4^+} \frac{x+5}{x-4} = +\infty$

$\lim_{x\to 4^-} f(x) = -\infty$



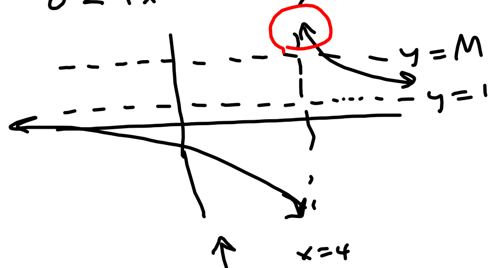
What's meant by $\lim_{x\to c} f(x) = \infty$?

It means you can make $f(x)$ arbitrarily large & positive by taking x sufficiently close to $x=c$.

Let M be given.

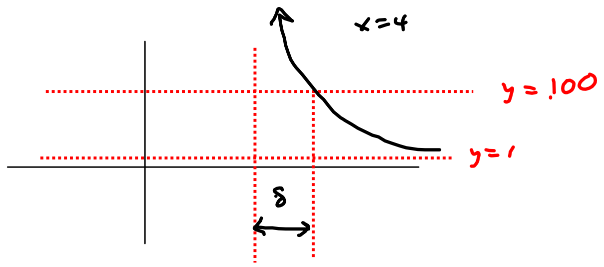
I can find a # δ so that if

$0 < |x-c| < \delta$, then $f(x) > M$



δ = Delta

Δ = Delta



Want $\frac{x+5}{x-4} > M$. Assume $x > 4$. Then $x-4 > 0$

$\Rightarrow x+5 > M(x-4)$

$\Rightarrow \frac{x+5}{M} > \underbrace{x-4}_{\delta}$ The kind of thing we'll be doing

$x+5 > Mx - 4M \longrightarrow$

$x+5 - Mx > -4M$

$x - Mx > -4M - 5$

$x(1-M) > -4M-5$

$x < \frac{-4M-5}{1-M}$ (We assume M is pretty big.)

$\underbrace{4 < x}_{\text{coming in from the right.}} < \frac{4M+5}{M-1}$

§1.6 Limit Laws.

NOTATION / (SHORTHAND)

$\lim_{x \to c} f(x) = \lim f$

Properties of Limits.

Assume $\lim_{x \to c} f(x) = L$ & $\lim_{x \to c} g(x) = M$

then

$\lim (f \pm g) = \lim f \pm \lim g = L \pm M$ "The limit of the sum or difference is the sum of the limits."

$\lim (fg) = (\lim f)(\lim g) = LM$

$\lim \left(\frac{f}{g}\right) = \frac{\lim f}{\lim g} = \frac{L}{M}$, provided $\lim g \neq 0$.

$\lim (f^n) = (\lim f)^n$

$\lim (\sqrt[n]{f}) = \sqrt[n]{\lim f}$, generally

(Watch out for $n = 2m$ & $\lim f$ negative)

If $f(x) < g(x)$, then

$\lim_{x \to c} f(x) \leq \lim_{x \to c} g(x)$.

$f(x) = \sin(x)$, $g(x) = 1$

Then if $0 < x < \frac{\pi}{2}$, then $\sin(x) < 1$

$\lim_{x \to \frac{\pi}{2}} \sin(x) = 1$ Squeeze theorem

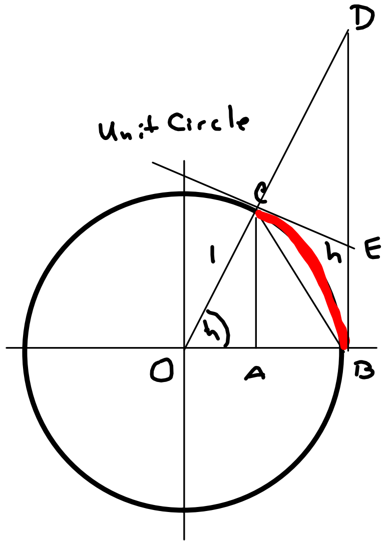
If $f(x) < g(x) < h(x)$ and $\lim f = \lim h$, then $\lim g = \lim f = \lim h$.

FACT: $\cos(x) < \frac{\sin(x)}{x} < 1$ for $0 < x < \frac{\pi}{2}$

$\Rightarrow \lim_{x \to 0} \cos(x) = 1$ & $\lim_{x \to 0} 1 = 1$

$\Rightarrow \lim_{x \to 0} \frac{\sin(x)}{x} = 1$

Looking ahead.



h = angle, in radians

$\frac{\sin h}{h} \xrightarrow{h \to 0} 1$

Assume $h > 0$

$\sin h = \frac{|AC|}{|OC|} = |AC| < |BC| < \text{arc } BC = h$

i.e., $\sin h < h$

$\Rightarrow \boxed{\frac{\sin h}{h} < 1} \quad \forall \; 0 < h < \frac{\pi}{2}$

Also, $h = \text{arc } DC < |BE| + |EC| < |BE| + |ED| = |BD|$

$h < |BD| = \frac{|BD|}{1} = \frac{|BD|}{|OB|} = \tan h$!

$\tan h = \frac{\sin h}{\cos h} > h$

$\Rightarrow \boxed{\frac{\sin h}{h} > \cos h}$

$\Rightarrow$

$\cos h < \frac{\sin h}{h} < 1$

$\downarrow_{h \to 0} \qquad \qquad \downarrow_{h \to 0}$

$1 \leq \lim_{h \to 0} \frac{\sin h}{h} \leq 1$

$\therefore \lim_{h \to 0} \frac{\sin h}{h} = 1$!

C2

Chapter 2.
Derivatives of Trig functions

#14 $\lim_{\theta \to 0} \frac{\sin(3\theta)}{\tan(4\theta)}$ Make sure you're in radians mode!

we "know" $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

$$\frac{\sin(3\theta)}{\tan(4\theta)} = \frac{\sin(3\theta)}{\frac{\sin(4\theta)}{\cos(4\theta)}} = \frac{\sin(3\theta)\cos(4\theta)}{\sin(4\theta)}$$

$$\frac{\sin(h)}{h} \xrightarrow{h \to 0} 1$$

$$= \frac{3}{3} \cdot \frac{4}{4} \cdot \frac{\theta}{\theta} \frac{\sin(3\theta)}{\sin(4\theta)} \cos(4\theta)$$

$$= \frac{3}{4} \left(\frac{\sin(3\theta)}{3\theta} \right) \left(\frac{4\theta}{\tan(4\theta)} \right) \cos(4\theta)$$

$$\xrightarrow{\theta \to 0} \frac{3}{4} \cdot 1 \cdot 1 \cdot 1 = \frac{3}{4}.$$

$$\begin{cases} x^2-3x+2 & \text{if } x \ge 3 \\ -4x+15 & \text{if } x<3 \end{cases}$$

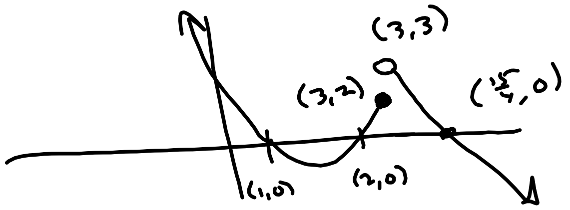
$x^2-3x+2 = (x-1)(x-2)$

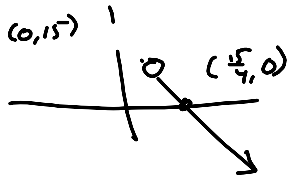
$x=3: \; 3^2-3(3)+2 = 2$

$-4x+15$ set $= 0$

$-4x = -15$

$x = \frac{15}{4}$

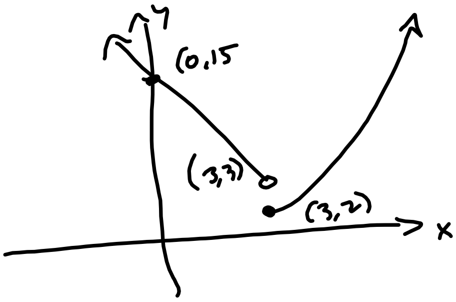




$(-4(3))+15$
$= -12+15$
$= 3$

No, it's not!

I just did $g(x) = \begin{cases} x^2-3x+2 & \text{if } x \le 3 \\ -4x+15 & \text{if } x > 3 \end{cases}$



This is what we want.

$$\lim_{x\to 2} \frac{x^2-3x+2}{x^2+3x-10} = \lim_{x\to 2} \frac{2^2-3(2)+2}{2^2+3(2)-10} = \frac{0}{0}\ ?!$$

If f is a polynomial, then $\lim_{x\to c} f(x) = f(c)$

$$\frac{x^2-3x+2}{x^2+3x-10} = \frac{(x-1)(x-2)}{(x+5)(x-2)} = \frac{x-1}{x+5} \xrightarrow{x\to 2} \frac{2-1}{2+5} = \boxed{\frac{1}{7} = \lim_{x\to 2} f(x)}$$

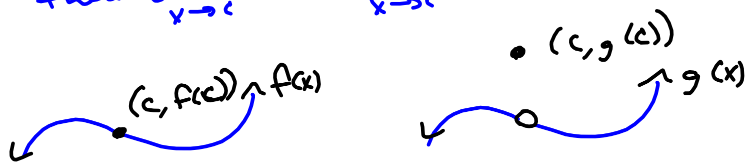
$x \neq 2$

$$\lim_{x\to 2} \frac{x^2-3x+2}{x^2+3x-10} = \lim_{x\to 2} \frac{(x-1)(x-2)}{(x+5)(x-2)}$$

$$= \lim_{x\to 2} \frac{(x-1)}{x+5} = \frac{2-1}{2+5} = \frac{1}{7}$$

If $f(x) = g(x)$, except possibly at $x=c$,

then $\lim_{x\to c} f(x) = \lim_{x\to c} g(x)$



This is why $\lim_{x\to 2} \frac{x^2-3x+2}{x^2+3x-2} = \lim_{x\to 2} \frac{x-1}{x+5}$