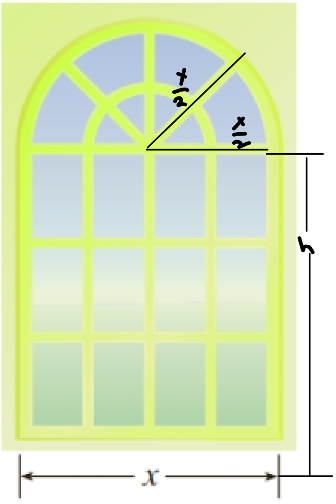
**Trig functions are periodic and that adds a wrinkle.  It also helps us model periodic stuff.**

S 1.1 #62

**62.** A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area $A$ of the window as a function of the width $x$ of the window.




Given: Perimeter of the window is 30 ft.

WANT to maximize the area

$A(x)$ = Area of Norman window (in $ft^2$) as a function of

$x$ = width of the window (ft)

Let $h$ = height of rectangular portion of the window.

Then Area $= A(x,h) = xh + \frac{\pi\left(\frac{x}{2}\right)^2}{2} = \frac{\pi x^2}{8} + xh$

b/c radius of $\frac{1}{2}$-circle is $\frac{x}{2}$, since $x$ = diameter.

$\pi\left(\frac{x}{2}\right)^2$ = area of full circle.

$A = A(x,h) = xh + \frac{\pi x^2}{8}$.

Need to be rid of $h$.

Perimeter $= 30$ → $h + x + h = 2h + X$

$= xh +$ area of $\frac{1}{2}$-circle $=$ → perimeter

$= xh + \frac{1}{2}$(area of circle)

$\cancel{= xh + \frac{1}{2}(\pi x)}$

$\cancel{= xh + \frac{\pi x}{2} = x\left(h + \frac{\pi}{2}\right) = 30}$

Area $= \pi D$, where $D$ = Diameter $= x$

$\cancel{\Rightarrow h + \frac{\pi}{2} = \frac{30}{x}}$

$\cancel{\Rightarrow h = \frac{30}{x} - \frac{\pi}{2}}$

$\cancel{\to A(x) = x\left(\frac{30}{x} - \frac{\pi}{2}\right) + \frac{\pi x^2}{8}}$

$\cancel{= 30 - \frac{\pi}{2}x + \frac{\pi x^2}{8}}$

$\cancel{= \frac{\pi}{8}x^2 - \frac{\pi}{2}x + 30}$

Perimeter

$= 2h + x + \pi \cdot \frac{x}{2}$

3 sides of the rectangle: Bottom & 2 sides.

Perimeter of $\frac{1}{2}$-circle.

$$= \cancel{\frac{\pi}{8}\left(x^2 - \frac{\pi}{2}\cdot\frac{8}{\pi}x \quad\right) + 30}$$

$$\cancel{= \frac{\pi}{8}\left(x^2 - 4x + 2^2 \quad\right) + 30 - \frac{\pi}{8}(4)}$$

$$\cancel{= \frac{\pi}{8}(x-2)^2 + \frac{60-\pi}{2}}$$

$$\cancel{= a(x-h)^2 + k}$$

~~where vertex $=(h,k) = \left(2, \frac{60-\pi}{2}\right)$~~

~~and $a = \frac{\pi}{8}$.~~

~~Scratch~~

$$\cancel{30 - \frac{\pi}{2}}$$

$$\cancel{= \frac{60-\pi}{2}}$$

Start over:

Perimeter $= 2h + x + \frac{\pi x}{2} = 30.$

Use this Auxiliary Equation to get rid of $h$.

$$\Longrightarrow 2h = 30 - x - \frac{x\pi}{2}$$

$$\Longrightarrow h = 15 - \frac{1}{2}x - \frac{\pi}{4}x$$

$$\Longrightarrow \text{Area} = \frac{\pi}{8}x^2 + xh$$

$$\boxed{= \frac{\pi}{8}x^2 + x\left(15 - \left(\frac{1}{2} + \frac{\pi}{4}\right)x\right)}$$

Good Setup

$$= \frac{\pi}{8}x^2 + 15x - \left(\frac{1}{2} + \frac{\pi}{4}\right)x^2$$

$$= \left(\frac{\pi}{8} - \frac{1}{2} - \frac{\pi}{4}\right)x^2 + 15x$$

$$= \left(-\frac{1}{2} - \frac{\pi}{8}\right)x^2 + 15x$$

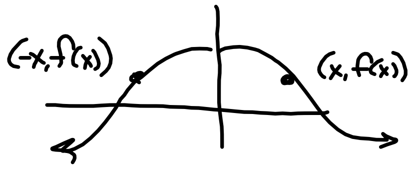
$$= -\left(\frac{1}{2} + \frac{\pi}{8}\right)x^2 + 15x$$

$$= -\left(\frac{4+\pi}{8}\right)x^2 + 15x$$

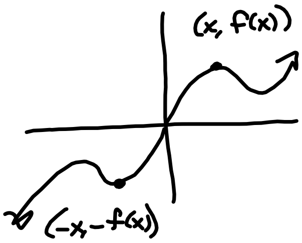
All cosmetic

Even, odd, or Neither

Even: $f(-x) = f(x)$




ODD: $f(-x) = -f(x)$




$x^2$ or any $x^{2n}, n \in \mathbb{Z}$.

(1) A even & B even $\Rightarrow$ A+B even, AB even, $\frac{A}{B}$ even

(2) A even & B odd $\Rightarrow$ A+B neither, AB ODD, $\frac{A}{B}$ ODD

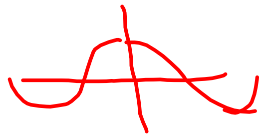
(3) A odd & B odd $\Rightarrow$ A+B odd, AB & $\frac{A}{B}$ even

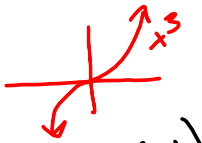
(1) $x^2 + x^4 + 1$ , $\cos(x) + 2$ ; $(x^2)(\cos(x))$ even

(2) $x^3 + x^2$ , $x^2 + \sin(x)$ ; $x^2 \sin(x)$ odd

(3) $x + x^3 + \sin(x) + \tan(x)$   $x^3 \sin(x)$ even

$\cos(-x) = \cos(x)$






$A(-x)+B(-x) = (-x)^2 + (-x)^4 + 1 = x^2 + x^4 + 1$

$A(-x)B(-x) = (-x)^2 \cos(-x) = x^2 \cos(x)$

$(-x)^2 + \sin(-x) = x^2 - \sin(x) \neq (f(x) \text{ or } f(x))$ Neither

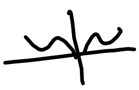
$(-x)^3 (\sin(-x)) = (-x^3)(-\sin(x)) = x^3 \sin(x) = f(-x)$

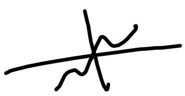
$f(x) = x^3 \sin(x) \Rightarrow$

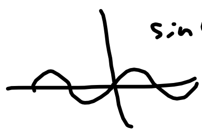
$f(-x) = (-x)^3 \sin(-x)$

$= (-x^3)(-\sin(x))$

**Graph of even and odd functions.**

even 

odd 



**Today: Section 1.3...**

**I have a review of Basic Functions and Transformations of Functions from College Algebra, as part of Week 6 Written Assignment for my 1340 students.**

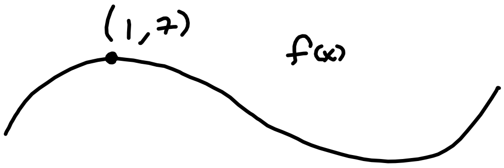
**College Algebra Review**
**Scroll down to #6 after**
**you CLICK HERE.**

**I notice that College Algebra and Calculus technically cover all the moves, but there's a real shortage of exercises that put all the moves into one exercise.**
**I think your Section 1.3 #11 does make use of all 4 moves:**
**Vertical stretch/shrink.**
**Horizontal stretch/shrink.**
**Horizontal shift (phase shift).**
**Vertical shift.**

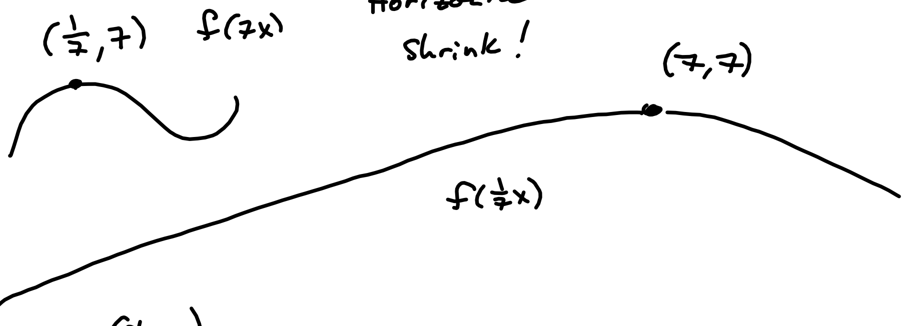
**Rules for transforming functions, with a square root example.**

1. $af(x)$    Vertical Shrink: $|a| < 1$    Vertical Flip
   $y \mapsto ay$    " Stretch: $|a| > 1$    $a < 0$

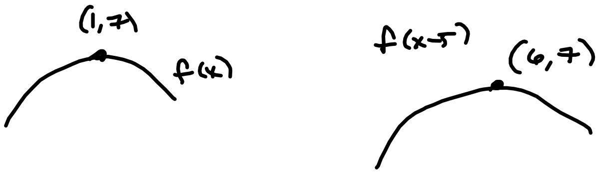
2. $f(bx)$    Horizontal Shrink: $|b| > 1$
   " Stretch: $|b| < 1$




Horizontal Shrink!

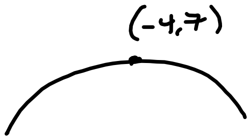



3. $f(x-c)$    (Phase) Horizontal Shift
   $f(x-5)$ RIGHT (Delay) $x \mapsto x+5$

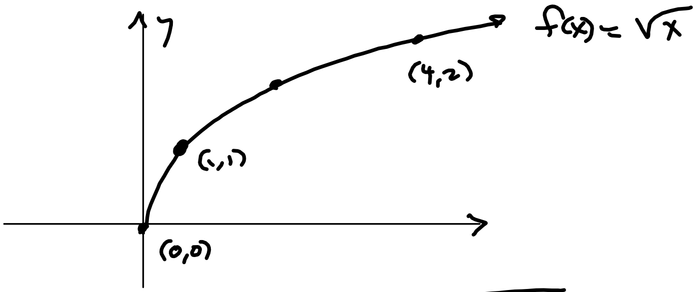



$f(x+5)$

Left 5!




4. $f(x)+d$    $f(x)+11$ up 11
   $f(x)-11$ down 11




Graph $g(x) = 3\sqrt{7x-21} - 14$

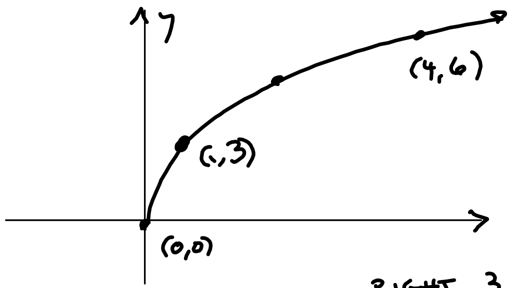
$= 3f(7x-21) - 14$

"That's fine as frog's hair!"
Victor Borge

0 Graph $f(x)$

1 $3 \cdot f(x) = 3\sqrt{x}$

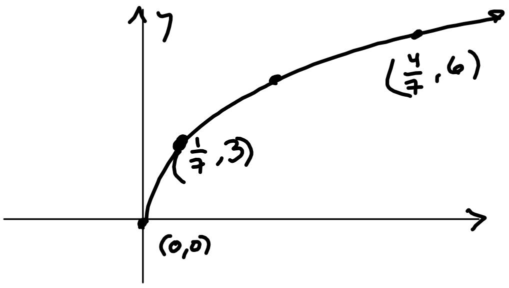
$y \mapsto 3y$



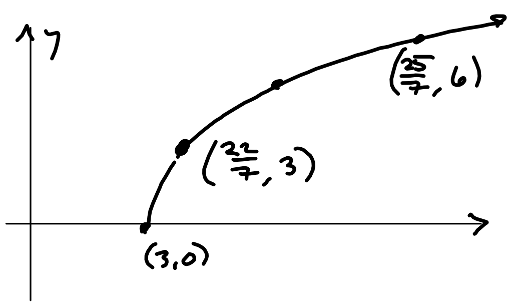

RIGHT 3

2 $7x-21 = 7(x-3)$

$3f(7x) = 3\sqrt{7x}$ $x \mapsto \frac{1}{7}x$



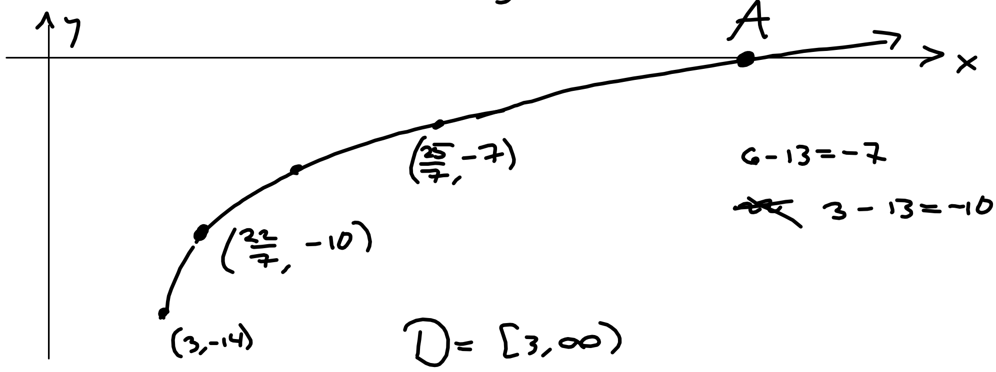

3 $3f(7(x-3))$ $x \to x+3$




$\frac{1}{7} + \frac{3}{1}\cdot\frac{7}{7} = \frac{22}{7}$

$\frac{4}{7} + \frac{21}{7} = \frac{25}{7}$

4 $g(x) = 3f(7(x-3)) - 14$ $\quad y \mapsto y - 14$




$6 - 13 = -7$

$3 - 13 = -10$

$D = [3, \infty)$

$R = [-14, \infty)$

x-int:

$g(x) \overset{SET}{=} 0$

$3\sqrt{7x-21} - 14 \overset{SET}{=} 0$

$3\sqrt{7x-21} = 14$

$\sqrt{7x-21} = \frac{14}{3}$

$7x - 21 = \left(\frac{14}{3}\right)^2$

$7x = \left(\frac{14}{3}\right)^2 + 21$

$\boxed{A = \left(\frac{\left(\frac{14}{3}\right)^2 + 21}{7}, 0\right)}$

$\boxed{x = \frac{\left(\frac{14}{3}\right)^2 + 21}{7}}$ IN A HURRY

$$\frac{\frac{196}{9} + \frac{21}{1} \cdot \frac{9}{9}}{7} = \frac{1}{7}\left[\frac{196 + 27}{9}\right] = \frac{223}{63} = x$$

$\longrightarrow$ x-int: $\left(\frac{223}{63}, 0\right)$

Attachments

260121.jpg

260121-excel.xlsx