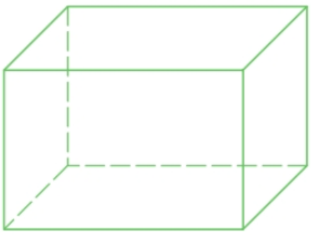
https://harryzaims.com/public_html/201/2410-spring-26/Grades/

EXAMPLE 5 A rectangular storage container with an open top has a volume of 10 m^3. The length of its base is twice its width. Material for the base costs \$10 per square meter; material for the sides costs \$6 per square meter. Express the cost of materials as a function of the width of the base.

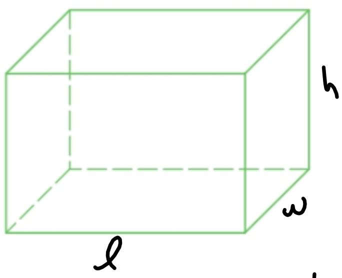
ℓ = length (m)
w = width (m)
h = height (m)
Volume = $\ell \cdot w \cdot h$ (m^3)

Given $\ell = 2w$
Cost is \$6/m^2 sides
& \$10/m^2 base.



we express the cost as a function of the width of the base.

Let $C = C(w)$ = cost of box as function of w = width.



Volume = $\ell w h = (2w)wh = 10\ m^3$ is given $\Rightarrow$ Auxiliary Equation

$\boxed{2w^2h = 10} \Rightarrow$

$\Rightarrow h = \frac{10}{2w^2} = \frac{5}{w^2}$ lowest terms

Cost $= \left(\frac{\$6}{m^2}\right)\left(\text{Area sides}\right) + \left(\frac{\$10}{m^2}\right)\left(\text{Area base}\right)$

$= (6)(2wh + 2\ell w) + (10)(\ell w)$

$= 6\left((2w)\left(\frac{5}{w^2}\right) + 2(2w)(w)\right) + 10\,(2w \cdot w)$

$= 6\left(\frac{10}{w} + 4w^2\right) + 10\,(2w^2)$

$= \frac{60}{w} + 24w^2 + 20w^2 = \boxed{\frac{60}{w} + 44w^2 \text{ dollars} = C(w)}$

§1.2 A Bunch of Basic Functions.

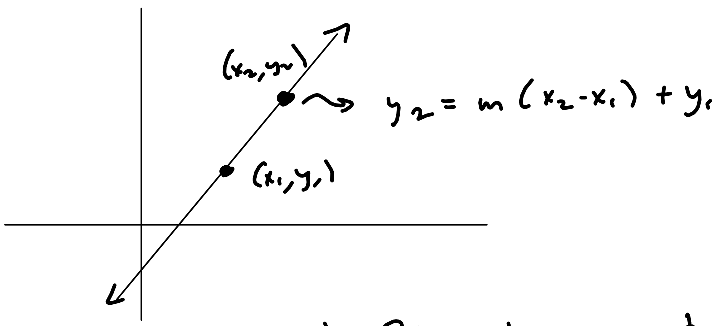
Lines: The line thru (x_1, y_1) with slope m is best represented in this way:

$$y = m(x - x_1) + y_1$$

You were taught $y - y_1 = m(x - x_1)$ for point-slope

LEARN $y = \underbrace{m(x - x_1)}_{\text{How much the height changes as we move away from } x = x_1} + y_1$

↑ The starting height

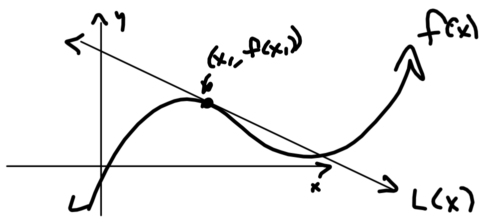


Tangent line to $f(x)$ at $x = x_1$ & m = the slope of $f(x)$ @ $x = x_1$

$m = f'(x_1) = m_{tan}$

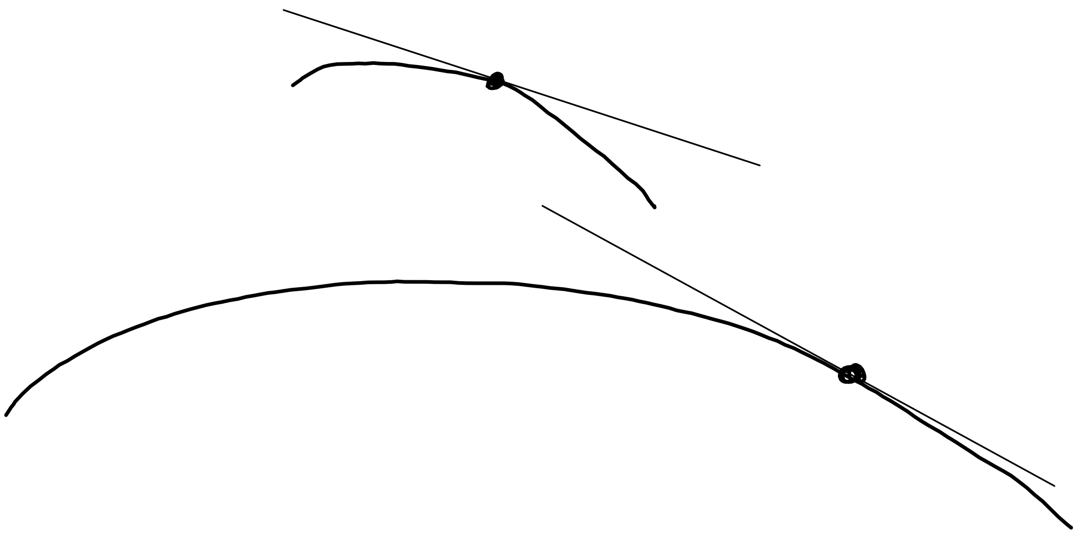
$L(x) = y = f'(x_1)(x - x_1) + f(x_1)$ = Linearization of $f(x)$

@ $x = x_1$

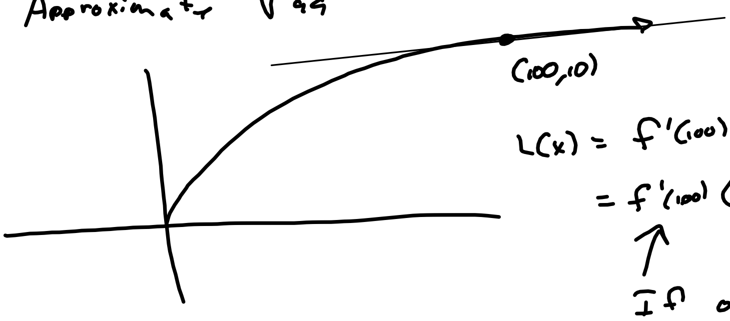


Smooth Functions are locally linear.

Tangent line is a great approxiimation close to *x*1.



Approximate $\sqrt{99}$



$$L(x) = f'(100)\,(x-100) + f(100)$$
$$= f'(100)\,(x-100) + 10$$

If only we knew the slope of $f(x)=\sqrt{x}$ @ $x=100$.

$$\lim_{h\to 0} \frac{f(100+h)-f(100)}{h} :$$

$(a-b)(a+b) = a^2-b^2$

$$\frac{\sqrt{100+h}-\sqrt{100}}{h} = \left(\frac{\overbrace{\sqrt{100+h}}^{a} - \overbrace{10}^{b}}{h}\right)\left(\frac{\sqrt{100+h}+10}{\underbrace{\sqrt{100+h}}_{a} + \underbrace{10}_{b}}\right)$$

$$= \frac{a^2-b^2}{h(a+b)} = \frac{\left(\sqrt{100+h}\right)^2 - 10^2}{h\left(\sqrt{100+h}+10\right)} = \frac{100+h-100}{h\left(\sqrt{100+h}+10\right)}$$

$$= \frac{\cancel{h}^{\,1}}{\cancel{h}_{\,1}\left(\sqrt{100+h}+10\right)} = \frac{1}{\sqrt{100+h}+10} \quad (h \neq 0) \xrightarrow{h\to 0} \frac{1}{\sqrt{100}+10} = \frac{1}{10+10} = \frac{1}{20}$$

$= \text{slope of } f(x)=\sqrt{x}$ @ $x=100.$

$$\Longrightarrow L(x) = f'(100)(x-100) + f(100)$$
$$= \frac{1}{20}(x-100) + 10$$

$$\longrightarrow \sqrt{99} \approx \frac{1}{20}(99-100)+10$$
$$= \frac{1}{20}(-1)+10$$
$$= -\frac{1}{20} + \frac{200}{20} = \boxed{\frac{199}{20} \approx \sqrt{99}}$$

$\sqrt{99}$ = 9.94987437107

$\frac{199}{20}$ = 9.95

Conjugate Trick.

$a+b$ & $a-b$

$a+\sqrt{b}$ & $a-\sqrt{b}$

$a+bi$ & $a-bi$

Quadratics

$f(x) = ax^2 + bx + c$

$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - a\left(\frac{b^2}{4a^2}\right)$

$\frac{\frac{b}{a}}{2} = \frac{b}{2a} \rightsquigarrow \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

Scratch

$c - \frac{b^2}{4a} = \frac{4ac - b^2}{4a}$

$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$

Vertex: $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

= Cheap way to complete the square.
(If you can't manipulate expressions the way I want you to...)

$f(x) = 2x^2 - 5x + 1$

$a = 2, b = -5, c = 1$

$-\frac{b}{2a} = -\left(\frac{-5}{2(2)}\right) = \frac{5}{4}$

$f\left(-\frac{b}{2a}\right) = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 1$

$= 2\left(\frac{25}{16}\right) - \frac{25}{4} + 1$

$= \frac{25}{8} - \frac{50}{8} + \frac{8}{8} = \frac{-17}{8}$

$\Rightarrow f(x) = 2\left(x + \frac{b}{2a}\right)^2 - \frac{17}{8}$

$= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$

$$f(x) = a\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a} \overset{\text{SET}}{=} 0 \longrightarrow$$

$$a\left(x+\frac{b}{2a}\right)^2 = -\frac{4ac-b^2}{4a} = \frac{b^2-4ac}{4a}$$

$$\sqrt{\left(x+\frac{b}{2a}\right)^2} = \sqrt{\frac{b^2-4ac}{4a^2}}$$

$$\left|x+\frac{b}{2a}\right| = \frac{\sqrt{b^2-4ac}}{2|a|}$$

$$x+\frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

$\sqrt{a^2} = |a|$

$\sqrt{3^2} = 3$

$\sqrt{(-3)^2} = 3$

Polynomials in general

$$a_n x^n + a_{n-1} x^{n-1} + \dots a_2 x^2 + a_1 x + a_0$$

$(0, a_0) =$ y-intercept

<u>End Behavior</u>:

a_n positive:

$n =$ even

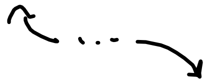
$3x^4 + \dots$

$n =$ odd

$3x^5 + \dots$

a_n negative

$-3x^{18}$



$-5x^{27}$

Factoring sums/differences of cubes

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

$f(x) = x^3$

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c} :$$

$$\frac{x^3 - c^3}{x - c} = \frac{\cancel{(x-c)}(x^2 + xc + c^2)}{\cancel{(x-c)}} = \underset{x \neq c}{x^2 + xc + c^2} \xrightarrow{x \to c}$$

$$c^2 + c^2 + c^2 = \boxed{3c^2}$$

$f'(c) = 3c^2$

$f'(x) = 3x^2$

Attachments

260121.jpg

260121-excel.xlsx