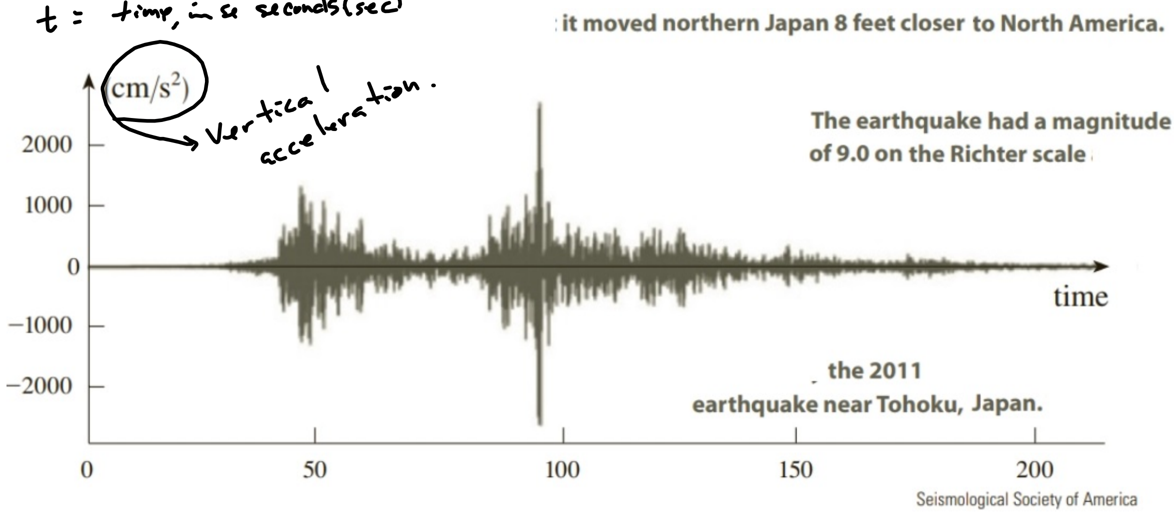
Functions

$f(t)$ = vertical acceleration in $\frac{cm}{sec^2}$ as a function of

$t$ = time, in ~~se~~ seconds(sec)




**I used Paint to sort of take measurements. People can kind of eyeball it, but I used Paint to estimate its range. Its domain is unclear, as is the horizontal scale, which I presume to be seconds, but how many seconds before the peak and after, I couldn't tell you.**

**It turns out not to matter, if your point is that things started shaking, with one great big shake that tapered off. It's just kind of unfortunate that there's no scale, because it doesn't tell you how long people were subjected to the Earth moving under their feet.**

**Domain and Range?**

D = ?

R = [-2700, 2700]

# Paint

- verbally (by a description in words) His heart beats faster the longer the runs.
- numerically (by a table of values) Pop. Data. Scatter Diagrams
- visually (by a graph) Graph of the seismograph
- algebraically (by an explicit formula) It costs \$5 per t-shirt.

Lexicon!

$H$ = Heartrate $= H(t)$ (Beats per minute) as function of
$t$ : time spent running (seconds)

$f(x) = x^2 + 2x + \sqrt{1-x}$
= Profit in \$ as a function of $x$ = # of units sold.

$P = P(t)$ = world pop as a function of
$t$ = the calendar year.

**Scatter diagram for table of data.**

**A *function* $f$ is a rule that assigns to each element $x$ in a set $D$ (for "Domain") exactly one element, called "$f(x)$," in a set $R$ (for "Range").**

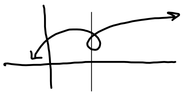
vertical line test

**$x$ is the input and $y = f(x)$ is the output. It's also called the *independent variable*.**

**$f$ is the machine that produces $y$ when you feed it $x$. It is called the *dependent variable*.**

**$D$ is the set of all acceptable (either from real-world model or resulting in a real output)**

**$R$ is the set of all outputs created by the action of $f$ on $D$.**



Not a function
Fails vertical line test.

$D = \mathcal{D} = \{x \mid f(x) \text{ is real}\}$ is the general rule

$= \{x \mid x \text{ is a possible input}\}$ based on the real-world setting.

$\mathcal{R} = \{y \mid y = f(x) \text{ for some } x \in \mathcal{D}\}$

In practice:
There are only 2 things stopping a real # $x$ from being in the domain of a function $f$ that accepts numbers as inputs:

For Domain,

1. $\frac{\text{STUFF}}{0}$ is Bad. For $f(x) = \frac{A}{B}$, solve $B \neq 0$

2. $\sqrt{\text{negative}}$ * is Bad. For $f(x) = \sqrt{A}$, solve $A \geq 0$

* Also $\sqrt[4]{\text{negative}}$ & $\sqrt[6]{\text{negative}}$ & ... $\sqrt[2n]{\text{negative}}$ Bad.
most elegant

1. $f(x) = \frac{70x}{1-x}$ $\Longrightarrow$ $D = \underbrace{\mathbb{R} \setminus \{1\}}_{\text{most elegant}} = \underbrace{(-\infty, 1) \cup (1, \infty)}_{\text{Interval}}$
$= \underbrace{\{x \mid x \neq 1\}}_{\text{set-builder}}$

Scratch:
Need: $1 - x \neq 0$
$1 \neq x$

$f(x) = \frac{27x}{x^2-3x+2}$

$D = \{x \mid x^2 - 3x + 2 \neq 0\}$

Method 1:
Solve $x^2 - 3x + 2 = 0$ & throw out the answers

$x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$
$\Longrightarrow$ $x - 2 = 0$ OR $x - 1 = 0$
$x = 2$ OR $x = 1$

Throw this out:
$D = \{x \mid x \neq 2 \text{ AND } x \neq 1\}$
$= \{x \mid x \neq 2 \text{ or } 1\}$

"NOT THIS AND NOT THAT"
= Not (THIS OR THAT)

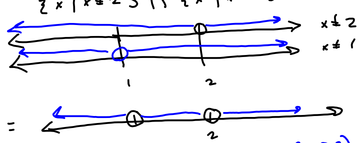
"$\sim$" means "not"

$\sim A$ AND $\sim B$ $\longleftrightarrow$ $\sim A \wedge \sim B$
means $\sim(A \text{ OR } B)$
means $\sim(A \vee B)$

NOT $A$ and NOT $B$
NOT $A$ $\cap$ NOT $B$
= NOT $(A \cup B)$

$\cap = \wedge$ = And = intersect
$\cup = \vee$ = or = Union

$\{x \mid x \neq 2\} \cap \{x \mid x \neq 1\}$




$= (-\infty, 1) \cup (1, 2) \cup (2, \infty)$

$x^2-3x+2 \neq 0$

$(x-2)(x-1) \neq 0$

$x-2 \neq 0$ AND $x-1 \neq 0$

$x \neq 2$ AND $x \neq 1$

which means $\{x \mid x \neq 2 \text{ and } x \neq 1\}$

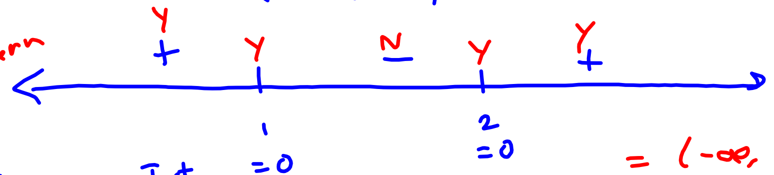
$= \{x \mid x \neq 2 \text{ or } 1\}$

$f(x) = \sqrt{x^2-3x+2}$

Need $x^2-3x+2 \geq 0$

$(x-2)(x-1) \geq 0$

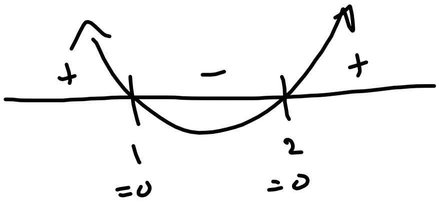
Sign Pattern




$\geq 0$

$= (-\infty, 1] \cup [2, \infty)$

| Int: | Tst | |
|---|---|---|
| $(-\infty, 1)$ | $0$ | $(0-1)(0-2) = (-1)(-2) = 2$ + yes |
| $(1,2)$ | $\frac{3}{2}$ | $(\frac{3}{2}-1)(\frac{3}{2}-2) = (\frac{1}{2})(-\frac{1}{2}) = -\frac{1}{4}$ − No |
| $(2,\infty)$ | $3$ | $(3-1)(3-2) = (2)(1) = 2$ + Yes |

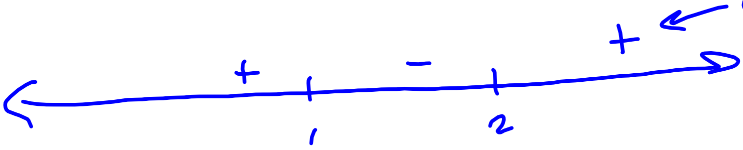


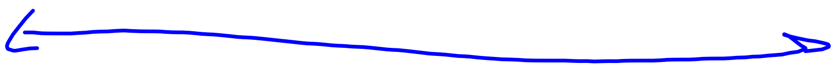

Want $\geq 0$

$(-\infty, 1] \cup [2, \infty)$

$(x-1)(x-2) = x^2 + \ldots$

End Behavior



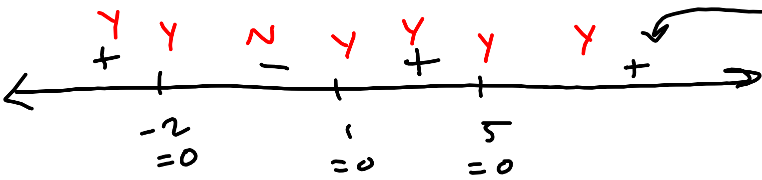



$\sqrt{(x-1)(x+2)^3(x-5)^2} = \sqrt{\text{stuff}}$

Need STUFF $\geq 0$

$x \cdot x^3 \cdot x^2 = x^6 + \ldots$

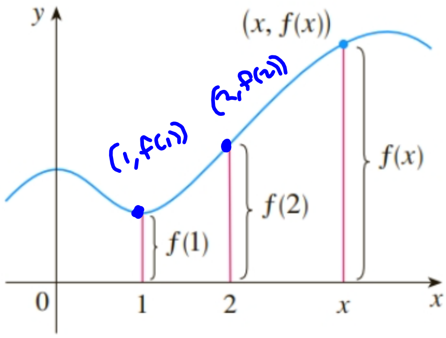
$x^6$

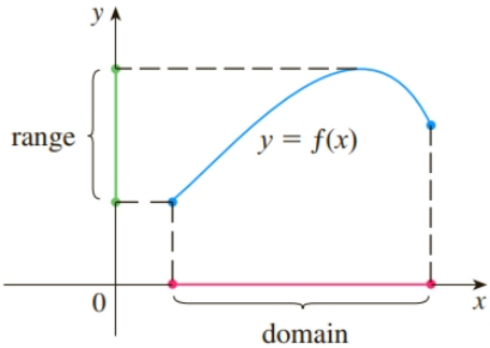



$\geq 0$

$= \underbrace{(-\infty, -2] \cup [1, 5] \cup [5, \infty)}$ 

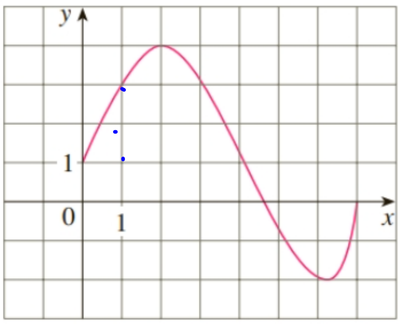
$[1, \infty)$




You kind of need the graph to find the range.

**EXAMPLE 1** The graph of a function $f$ is shown in Figure 6.
(a) Find the values of $f(1)$ and $f(5)$.
(b) What are the domain and range of $f$?




$f(1) = 3$
$f(5) \approx -.7$ ish.

$D = [0, 7]$
$R = [-2, 4]$

## Graph and find domain and range

### Lines (Linear Functions)

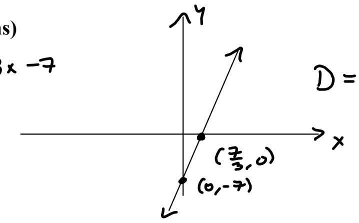
$y = 3x - 7$

| $x$ | $y$ |
|---|---|
| $0$ | $-7$ |
| $\frac{7}{3}$ | $0$ |

$3x - 7 = 0$

$3x = 7$

$x = \frac{7}{3}$

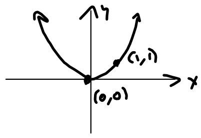



$D = \mathbb{R} = (-\infty, \infty)$

$R = (-\infty, \infty)$

### Quadratic Functions (Quick Complete-the-Square?)

$f(x) = x^2$, $f(x) = 2x^2 - 5x + 1$




$D = \mathbb{R}$

$R = [0, \infty)$

$2x^2 - 5x + 1$

$= 2(x^2 - \frac{5}{2}x + (\frac{5}{4})^2) + 1 - 2(\frac{25}{16})$

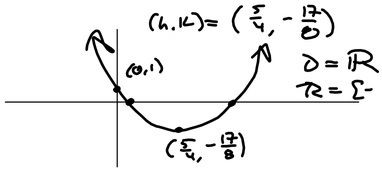
$\frac{5}{2} \div 2 = \frac{\frac{5}{2}}{2} = \frac{5}{4}$

$\frac{8-25}{8} = -\frac{17}{8}$

$= 2(x - \frac{5}{4})^2 + 1 - \frac{25}{8}$

$= 2(x - \frac{5}{4})^2 - \frac{17}{8}$

$(h, k) = (\frac{5}{4}, -\frac{17}{8})$




$D = \mathbb{R}$

$R = [-\frac{17}{8}, \infty)$

### Difference Quotient at *a* or just do it at *x*, why don't ya?

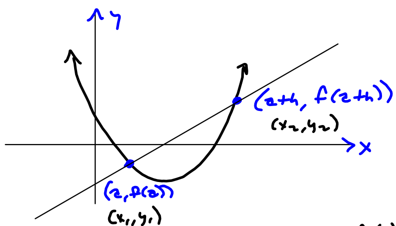
The book calculates $f(a + h)$, $f(a)$, separately, simplifies their difference and *then* plugs it into the difference quotient.

Find $\frac{f(a+h) - f(a)}{h}$ for $f(x) = 2x^2 - 5x + 1$

SPEED MOVE

$(x+y)^2 = x^2 + 2xy + y^2$

$(x-y)^2 = x^2 - 2xy + y^2$




$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(a+h) - f(a)}{a+h-a}$

$= \frac{f(a+h) - f(a)}{h}$

$f(x) = 2x^2 - 5x + 1$

$f(a) = 2a^2 - 5a + 1$

BOOK DOES THESE Separately.

$f(a+h) = 2(a+h)^2 - 5(a+h) + 1$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+h)^2 = x^2 + 2xh + h^2$

$(a+h)^2 = a^2 + 2ah + h^2$

My way:

$\frac{f(a+h) - f(a)}{h} = \frac{2(a+h)^2 - 5(a+h) + 1 - (2a^2 - 5a + 1)}{h}$

$= \frac{2(a^2 + 2ah + h^2) - 5a - 5h + 1 - 2a^2 + 5a - 1}{h}$

$= \frac{2a^2 + 4ah + 2h^2 - 5a - 5h - 2a^2 + 5a}{h}$

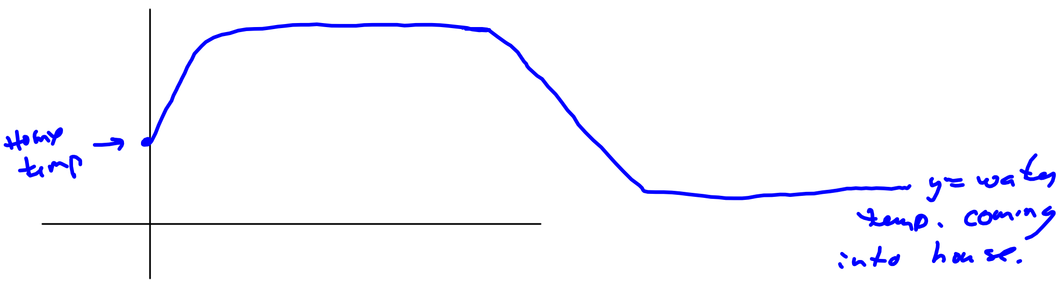
$= \frac{4ah + 2h^2 - 5h}{h} = \frac{h(4a + 2h - 5)}{h} = 4a + 2h - 5$

$(h \neq 0)$

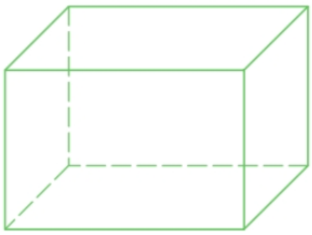
$\xrightarrow{h \to 0} 4a - 5 = \text{slope @ } x = a!$

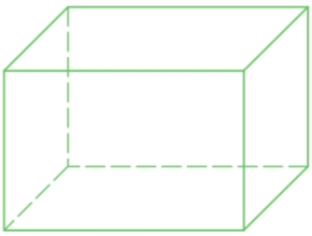
**Let $T$ = Temp of hot water coming out of the faucet, as a function of $t$ = # of minutes the faucet is running.**

**(Only works for hot water heaters with a big tank. Not for modern, on-demand hot water...)**



**EXAMPLE 5** A rectangular storage container with an open top has a volume of 10 $m^3$. The length of its base is twice its width. Material for the base costs \$10 per square meter; material for the sides costs \$6 per square meter. Express the cost of materials as a function of the width of the base.





Attachments

260121.jpg

260121-excel.xlsx