MAT 2410 Week 14 Written Assignment
75 points Covers Section 5.1, 5.2, 6.1 – 6.4

5.1 – Area Between Curves
5.2 – Volumes by the Disk Method
6.1 – Inverse Functions
6.2 – Exponential Functions and their Derivatives
6.3 – Logarithmic Functions
6.4 – Derivatives of Logarithmic Functions

1. (5 pts) Let $f(x)=2x^3-7x^2+5x$ and $g(x)=x^3-x^2-3x$. Find the area of the region bounded by $f(x), g(x), x=0,$ and $x=5.$

2. Consider the region bounded by $y=(x-1)^3$, $y=0,$ $x=1,$ and $x=2$.

   a. (5 pts) Sketch this region and find its area.

   b. (5 pts) Sketch the solid obtained by rotating this region about the line $y=3$. Include a representative washer on your graph and write the integral for finding its volume by the disk method.

   c. (5 pts) Evaluate the integral from part b.

   d. (5 pts) Sketch the solid obtained by rotating this region about the line $x=-3$. Include a representative washer on your graph and write the integral for finding its volume by the washer method.

   e. (5 pts) Evaluate the integral from part d.

3. The function $f(x)=x^2+5x+11$ is 1-to-1 on the restricted domain $D=\left[-\frac{5}{2},\infty\right)$.

   a. (5 pts) Find the inverse function $f^{-1}(x)$. State its domain and range. Plug in $x = 5$ to determine $f^{-1}(5)$

   b. (5 pts) Find $\left(f^{-1}\right)'(x)$ by differentiating your answer for part a. Plug in $x = 5$ to determine $\left(f^{-1}\right)'(5)$.

   c. (5 pts) Find $\left(f^{-1}\right)'(x)$ by applying a theorem about derivatives of inverse functions. Then find $\left(f^{-1}\right)'(5)$, using this result.

4. Differentiate each of the following:

   a. (5 pts) $y=7\cdot 6^{x^2-3x}$

b. (5 pts) $y = \ln\left(\dfrac{\sqrt[5]{x^2 - 3x}}{\left(3x^5 + 5x\right)^3}\right)$

c. (5 pts) $y = \left[\sin\left(x\right)\right]^{x^2 - 3x}$ (Logarithmic Differentiation)

d. (5 pts) $y = \left(x^4 - 7x^2\right)^5 \left(\sin^8\left(x\right)\right)$ (Use Logarithmic Differentiation)

e. (5 pts) $y = \ln\left(x^2 - 3x\right)$

f. (5 pts) $y = \log_7\left(x^2 - 3x\right)$

5. (**Bonus 5 pts**) Evaluate the indefinite integral: $\int \cot\left(x\right) dx$