MAT 2410 Week 13 Written Assignment
50 points Covers Section 4.1 – 4.5

4.1 - Areas and Distances
4.2 - The Definite Integral
4.3 - The Fundamental Theorem(s) of Calculus
4.4 - Indefinite Integrals and Net Change Theorem
4.5 - u-Substitution and Net Change Thm

1. Suppose $f(x)$ is a continuous, nonnegative function on the closed interval $[0,5]$. Let $m = \min_{x\in[0,5]}\{f(x)\}$ and $M = \max_{x\in[0,5]}\{f(x)\}$.

   a. (5 pts) Give an upper and lower estimate for $\int_0^5 f(x)\,dx$.

   b. (5 pts) Draw a sketch describing this situation.

2. (5 pts) Calculate the definite integral $\int_{-2}^{4}(3x^2+2x-5)\,dx$ by the limit definition of the definite integral.

3. (5 pts) Use the 2nd Fundamental Theorem of Calculus to evaluate $\int_{-1}^{4}\left(x^2-\pi\sin\left(\frac{\pi}{6}x\right)\right)dx$.

4. Use the 1st Fundamental Theorem of Calculus to evaluate each of the following derivatives:

   a. (5 pts) $\frac{d}{dx}\left[\int_0^x\left(t^2-\pi\sin\left(\frac{\pi}{6}t\right)\right)dt\right]$

   b. (5 pts) $\frac{d}{dx}\left[\int_0^{\sqrt{3x}+\sin(x)}\left(t^2-\pi\sin\left(\frac{\pi}{6}t\right)\right)dt\right]$

5. Evaluate the following definite and indefinite integrals

   a. (5 pts) $\int(2x-3)\sin(x^2-3x+5)\,dx$

   b. (5 pts) $\int_{-3}^{3}\frac{\sin(x)}{x^4+5x^2+1}dx$

   c. (5 pts) $\int_{-3}^{3}x\sqrt{x+4}\,dx$

d. (5 pts) $\int \sec^4(x)\tan(x)\,dx$

6. (**Bonus** 5 pts) Evaluate $\int_{-5}^{5}(x+7)\sqrt{25-x^2}\,dx$ by splitting it into 2 integrals and interpreting one of the integrals as an area.