MAT 2410 Week 8 Written Assignment
50 points Covers Section 3.1 – 3.3

3.1 – Max and Min Values
3.2 – The Mean Value Theorem
3.3 – How Derivatives Affect the Shape of a Graph

1. Let $f(x) = 2x^3 - 27x^2 + 84x$.
   a. (5 pts) Convince me that there is a point $c \in [0,3]$ such that $f'(c)$ is the same as the average slope, m_{avg}, of f on the interval $[0,3]$, *without finding c, itself*!
   b. (5 pts) What is the average slope, m_{avg}, of f on the interval $[0,3]$? What is $f'(x)$? Find *c*.

2. Let $f(x) = (x+2)^2(x-1)^3$.
   a. (5 pts) Find the absolute maximum and minimum of f on the interval [– 3, 0].
   b. (5 pts) Find the *open* intervals on which f is increasing. Find the open intervals on which f is decreasing.
   c. (5 pts) Find the open intervals on which f is concave up. Find the open intervals on which f is concave down.
   d. (5 pts) Use all the information from parts a – d to sketch the graph of f. Label all intercepts, max/min points, and inflection points. You may put the ordered-pair labels directly on the graph or make a legend/key as I will demonstrate in lecture.

3. Let $f(x) = x^2\sqrt{-x^2+5x-6}$.
   a. (5 pts) What is the domain of f?
   b. (5 pts) Use a graphing utility to sketch the graph of f. Include all max/min values and intercepts. Round answers to 2 decimal places.
   c. (**Bonus 5 pts**) Use calculus to find the *exact* maximum value. What is the range of f?

4. (5 pts) Let $f(x) = x^2(x-3)^{\frac{3}{5}}$. Sketch the graph of f. Clearly label all *x*- and *y*-intercepts, local max/min points, and inflection points. Each label should be an ordered pair or a letter referring to an ordered pair in a key or legend for the sketch. It's vital that your sketch capture the main features and shape.

5. (5 pts) Sketch the graph of $f(x) = \cos(x)$ on the interval $[0, 2\pi]$. Show all intercepts, extrema, and inflection points. The curvature of your sketch should match the results of your supporting work.