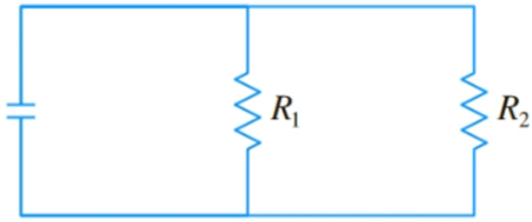
2410 WEEK 7 ASSIGNMENT SOL'NS H. MILLS

1. (10 pts) If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure on the right, then the total resistance R, measured in ohms (Ω), is given by



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If R_1 and R_2 are increasing at rates of 0.4 Ω/s and 0.1 Ω/s, respectively, how fast is R changing when $R_1 = 90\ \Omega$ and $R_2 = 70\ \Omega$?

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Longrightarrow$$

$$R^{-1} = R_1^{-1} + R_2^{-1} \Longrightarrow$$

$-R^{-2}R' = -R_1^{-2}R_1' - R_2^{-2}R_2'$. We supply the known values and solve for, but first, clean it up, some:

$$\frac{R'}{R^2} = \frac{R_1'}{R_1^2} + \frac{R_2'}{R_2^2} \Longrightarrow$$

$$\frac{R'}{R^2} = \frac{.4}{90^2} + \frac{.1}{70^2}$$

We still don't have R, but

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{90} + \frac{1}{70} = \frac{1}{90}\cdot\frac{7}{7} + \frac{1}{70}\cdot\frac{9}{9}$$

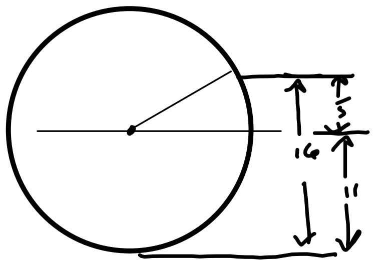
$$= \frac{7+9}{630} = \frac{16}{630} = \frac{8}{315} = \frac{1}{R} \Longrightarrow R = \frac{315}{8}$$

$$\frac{R'}{\left(\frac{315}{8}\right)^2} = \frac{.4}{8100} + \frac{.1}{4900}.$$

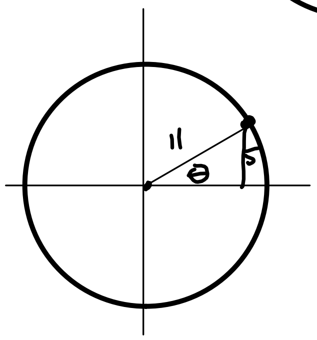
$$R' = \left(\frac{315}{8}\right)^2\left(\frac{.4}{8100} + \frac{.1}{4900}\right) = \boxed{\frac{277}{2560} \approx .1082031250\ \frac{\Omega}{s}}$$

2410 W#7 H. MILLS

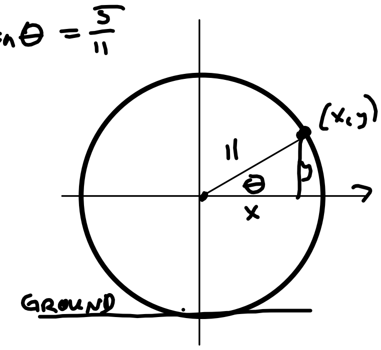
(2) (10 pts) **A Ferris wheel with a radius of 11 m is rotating at a rate of one revolution every 2 minutes. How fast is a rider rising when his seat is 16 m above the ground?**



Use hub of wheel as origin.



$\sin\theta = \frac{5}{11}$



y = vertical height, measured from the x-axis.

$\frac{y}{11} = \sin\theta$

$y = 11\sin\theta$

$\frac{dy}{dt} = 11\cos(\theta)\cdot\frac{d\theta}{dt}$

We want $\left.\frac{dy}{dt}\right|_{y=5}$

We need $\frac{d\theta}{dt}$:

$\left(\frac{\text{One rev}}{2\text{ min}}\right)\left(\frac{2\pi\text{ radians}}{\text{One rev}}\right) = \frac{\pi\text{ radians}}{\text{min}}$ or just $\frac{\pi}{\text{min}}$

Then $\left.\frac{dy}{dt}\right|_{y=5} = 11\cos\theta\,\frac{d\theta}{dt}$

Need θ @ y=5, too.

$\theta = \arcsin\left(\frac{5}{11}\right) \longrightarrow$

$\sqrt{11^2-5^2}$

$=\sqrt{121-25}$

$=\sqrt{96}$

$=4\sqrt{6}$

2 | 96
2 | 48
2 | 24
2 | 12
2 | 6
3

$\left.\frac{dy}{dt}\right|_{y=5} = \cos\left(\arcsin\left(\frac{5}{11}\right)\right)\cdot\pi$

$= \frac{4\sqrt{6}}{11}\cdot\pi = \frac{4\pi\sqrt{6}}{11}\ \frac{\text{m}}{\text{min}} \approx$

$\approx \boxed{2.798290539\ \frac{\text{m}}{\text{min}} \approx \frac{dy}{dt}}$

2410 MILLS

3) $f(x) = \sqrt{x+2} = (x+2)^{\frac{1}{2}}$

a) 5pts we find eq'n of tangent line to $f(x)$

@ $(x_1, y_1) = (7,3)$

$f(x_1) = 3 = f(7)$

$f'(x) = \frac{1}{2}(x+2)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x+2}}$

$f'(x_1) = f'(7) = \frac{1}{2\sqrt{7+2}} = \frac{1}{2(3)} = \frac{1}{6}$

$L(x) = f'(x_1)(x - x_1) + f(x_1)$

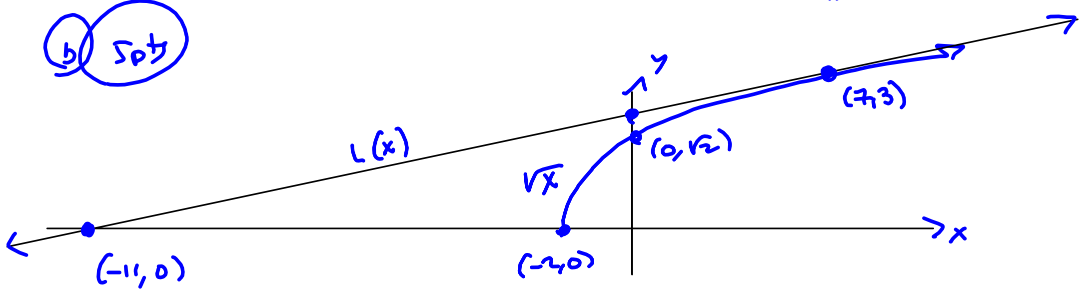
$= \boxed{y = \frac{1}{6}(x-7) + 3 = L(x)}$

$\frac{1}{6}(x-7) + 3 = 0$

$x - 7 = -18$

$x = -11$

b) 5pts



2410 MI-LS

(4) Painters paint the sides & top of the outside of a cylindrical water tank with radius $r = 5$ ft and height $h = 10$ ft. We find the volume of paint needed for a 0.006 in coat of paint in 2 ways:

(a) (5pts) Direct Calculation (ft^3)

Let V = volume of the cylinder. Then adding a coat of paint .006 in thick means the volume increases to that of a cylinder with $r = 5\text{ ft} + (.006\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)$ & $h = 10\text{ ft} + (.006\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)$

I think we need h in terms of r:

$V(r,h) = \pi r^2 h$

$V(r+\Delta r, h+\Delta h) - V(r,h)$

$= \pi\left(5 + \frac{.006}{12}\right)^2\left(10 + \frac{.006}{12}\right) - \pi(5^2)(10) = \frac{500040001\pi}{8000000000}$

$\approx$ 0.1963652 ft^3

(b) (5pts) with a differential.

Need one variable. Hmmmm. I didn't think about that

Maybe we can patch it.

when $r=5, h=10$, $V = \pi(5)^2(10) = 250\pi$

$250\pi = \pi r^2 h \rightarrow h = \frac{250}{r^2}$ No help.

No patch.

No questions were asked. No curiosity, I guess.

This question is porly posed. You can do part a, but part b? Not so much. Not in a course in single-variable calculus.

2410 MILLS

5 5pts We estimate $\sin(33^\circ)$

$33^\circ = 30^\circ + 3^\circ = \frac{\pi}{6} + \frac{3\pi}{180} = \frac{\pi}{6} + \frac{\pi}{60}$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$

$f'(\frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$f(\frac{\pi}{6}) = \sin(\frac{\pi}{6}) = \frac{1}{2}$

$L(x) = f'(\frac{\pi}{6})(x - \frac{\pi}{6}) + f(\frac{\pi}{6})$

$= \frac{\sqrt{3}}{2}(x - \frac{\pi}{6}) + \frac{1}{2}$

$\Rightarrow L(\frac{\pi}{6} + \frac{\pi}{60}) = \frac{\sqrt{3}}{2}(\frac{\pi}{6} + \frac{\pi}{60} - \frac{\pi}{6}) + \frac{1}{2}$

$= \frac{\sqrt{3}}{2}(\frac{\pi}{60}) + \frac{1}{2} = \boxed{\frac{\sqrt{3}\pi + 60}{120} \approx \sin(33^\circ) \approx 0.5453449841}$

Differential:

$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$

$= \sin(\frac{\pi}{6}) + \frac{\sqrt{3}}{2}(\frac{\pi}{60}) = \frac{\sqrt{3}\pi}{120} + \frac{1}{2}$ ✓

same.

Calculator result: 0.5446390350