2410 WEEK 05 SOLNS H. MILLS

① 5pts Let $f(x) = x^2+3x-7$. We apply the rules of differentiation.

ⓐ Differentiate $f(x)$ w.r.t. x:

$$\boxed{\frac{df}{dx} = 2x+3}$$

ⓑ differentiate $f(x)$ w.r.t. z

$$\boxed{\frac{df}{dz} = 0}$$

ⓒ $\boxed{\frac{dy}{dx} = \frac{df}{dx} = 2x+3}$

② 5pts we find the eq'n of the tangent line $L_{-2}(x)$ to $f(x)$ @ $x=2$.

$L(x) = f'(x_1)(x-x_1) + f(x_1)$

$f'(x) = 2x+3 \Rightarrow f'(x_1) = f'(-2) = 2(-2)+3 = -1$

$f(x_1) = f(-2) = (-2)^2+3(-2)-7 = 4-6-7 = 4-13 = -9$

$$\Rightarrow \boxed{L(x) = -1(x-(-2)) - 9}$$

③ 5pts we sketch $f(x)$ & $L(x) = L_{-2}(x)$ on same set of coordinate axes with labels $A, B, C, \ldots$ from left to right

$f(x) = x^2+3x-7 = x^2+3x+\left(\frac{3}{2}\right)^2 - \frac{9}{4} - \frac{28}{4}$

$= \left(x+\frac{3}{2}\right)^2 - \frac{37}{4} \longrightarrow (h,k) = \left(-\frac{3}{2}, -\frac{37}{4}\right) = \text{vertex}$

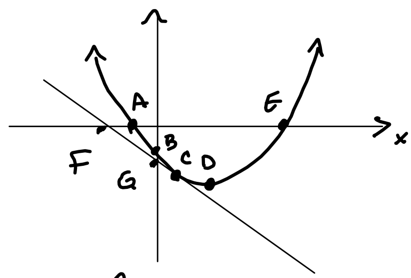
$(-2,-9) = (x_1, f(x_1))$

x-int:

$\left(x+\frac{3}{2}\right)^2 = \frac{37}{4}$

$x = \frac{-3 \pm \sqrt{37}}{2}$

y-int: $(0,-7)$



$F = (-11, 0)$

$G = (0, -11)$

$A = \left(\frac{-3-\sqrt{37}}{2}, 0\right)$

$B = (0,-7)$

$C = (-2,-9)$

$D = \left(-\frac{3}{2}, -\frac{37}{4}\right)$

$E = \left(\frac{-3+\sqrt{37}}{2}, 0\right)$

I guess I left off the x- & y-ints of $L(x)$:

$y = -(x+2)-9$

$= -x-2-9$

$= -x-11$

x	y
0	-11
-11	0

$-x-11 = 0$

$x = -11$

2410 W#5 MILLS

(4) **We differentiate the following. We do not simplify.1**

(a) (5pts) $F(y) = \left(\frac{1}{y^2} + \frac{2}{y^5}\right)(y^3 - y^7)$

$= (y^{-2} + 2y^{-5})(y^3 - y^7) \Longrightarrow$

$$\boxed{F'(y) = (-2y^{-3} - 10y^{-6})(y^3 - y^7) + (y^{-2} + 2y^{-5})(3y^2 - 7y^6)}$$

(b) (5pts) $g(\theta) = \theta^2 \cos\theta \Longrightarrow \boxed{g'(\theta) = 2\theta\cos\theta - \theta^2 \sin\theta}$

(c) (5pts) $H(x) = \frac{x^2 + 3x - 7}{x^3 + 1} \Longrightarrow$

$$\boxed{H'(x) = \frac{(2x+3)(x^3+1) - (x^2+3x-7)(3x^2)}{(x^3+1)^2}}$$

(d) (5pts) $Q(\omega) = \frac{\omega^2 \tan(\omega)}{\cos(\omega) + \omega} \Longrightarrow$

$$\boxed{Q'(\omega) = \frac{\left(2\omega\tan(\omega) + \omega^2 \sec^2(\omega)\right)\left(\cos(\omega) + \omega\right) - \left(\omega^2 \tan(\omega)\right)\left(-\sin(\omega) + 1\right)}{\left(\cos(\omega) + \omega\right)^2}}$$

2410 W#5 MILLS

(5) (5pts) Find eq'n of tangent line to $y=\frac{5x}{x^2-4}=f(x)$ @ $x_1=3$

$$f'(x)=\frac{5(x^2-4)-5x(2x)}{(x^2-4)^2} \Rightarrow$$

$$f'(3)=\frac{5(9-4)-5(3)(2(3))}{(9-4)^2}=\frac{5(5)-15(6)}{5^2}$$

$$=\frac{5(5-3(6))}{5^2}=\frac{5-18}{5}=-\frac{13}{5}=m_{tan}$$

$$f(3)=\frac{5(3)}{3^2-4}=\frac{15}{9-4}=\frac{15}{5}=3 \to (x_1,y_1)=(x_1,f(x_1))=(3,3)$$

$$L(x)=y=m_{tan}(x-x_1)+y_1$$

$$=f'(3)(x-3)+f(3)$$

$$=\boxed{-\frac{13}{5}(x-3)+3=L(x)=y}$$

(6) $\boxed{\text{5pts Bonus}}$ $f(x)=\frac{5x}{x^2-4}$

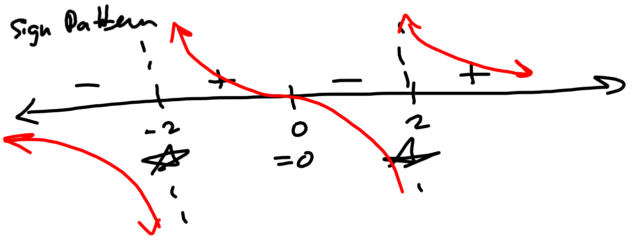
$\mathcal{D}=\mathbb{R}\setminus\{\pm2\}$

$\boxed{x=2,\ x=-2 \text{ are V.A.}}$

H.A: f is proper $\Rightarrow$ $\boxed{y=0 \text{ is H.A.}}$

$$f(x)=\frac{5x}{x^2-4}\overset{SET}{=}0 \Rightarrow 5x=0 \Rightarrow x=0 \to (0,0)$$

Sign Pattern



$f(x)=\frac{5x}{x^2-4}$