2410 WRITTEN WORK WEEK 4 SOL'NS 4. MILLS

(1) $f(x) = \sqrt{x+2}$

(a) (5pts) Find slope, m_{tan}, of the tangent line to f @ $x = a > -2$

$$\frac{f(a+h) - f(a)}{h} = \left(\frac{\sqrt{a+h+2} - \sqrt{a+2}}{h}\right)\left(\frac{\sqrt{a+h+2} + \sqrt{a+2}}{\sqrt{a+h+2} + \sqrt{a+2}}\right)$$

$$= \frac{1}{h}\left(\frac{a+h+2-(a+2)}{\sqrt{a+h+2}+\sqrt{a+2}}\right) = \frac{h}{h\left(\sqrt{a+h+2}+\sqrt{a+2}\right)}$$

$$= \frac{1}{\sqrt{a+h+2}+\sqrt{a+2}} \xrightarrow{h \to 0} \frac{1}{\sqrt{a+2}+\sqrt{a+2}}$$

$(h \neq 0)$

$$= \boxed{\frac{1}{2\sqrt{a+2}} = m_{tan} = f'(a)}$$

(b) (5pts) $L(x) = y = f'(a)(x-a) + f(a)$ is tangent line to f @ a. Let $a = 2$.

$L(x) = f'(2)(x-2) + f(2)$

(Scratch: $f(2) = \sqrt{2+2} = \sqrt{4} = 2 = f(2)$

$f'(2) = \frac{1}{2\sqrt{2+2}} = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \boxed{\frac{1}{4} = f'(2)}$)

$$= \boxed{\frac{1}{4}(x-2) + 2 = L(x)}$$

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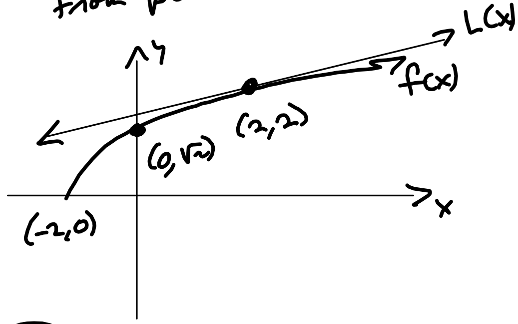
c 5pts We sketch $f(x)$ and the tangent line from part b.



d 2pts $f(2.1) \approx L(2.1) = \frac{1}{4}(2.1-2)+2$

$= \frac{1}{4}(.1)+2$

$= .25(.1)+2$

$= .025+2$

$\boxed{= 2.025 = L(2.1) \approx f(2.1) \approx \sqrt{2.1}}$

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MILLS

(3) The cost $C(x)$, in dollars,
of producing x units of a commodity, is given by
$$C(x) = 0.1x^2 + 11x + 7000$$

(a) (5pts) The average rate of change of cost, $C(x)$ on the interval $[100, 101]$ is

$$\frac{C(101) - C(100)}{101 - 100} =$$

$$\frac{.1(101)^2 + 11(101) + 7000 - (.1(100)^2 + 11(100) + 7000)}{1}$$

$$= .1(101)^2 + 11(101) - .1(100)^2 - 11(100)$$

$$= .1(101^2 - 100^2) + 11(101 - 100)$$

$$= .1(101 + 100)(101 - 100) + 11$$

$$= .1(201)(1) + 11 = 20.1 + 11 = 31.1 \frac{\$}{\text{unit produced}}$$

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(b) 5pts Find $C'(100)$ = the instantaneous rate of change of cost of producing x units, when $x = 100$.

$$\frac{C(x) - C(100)}{x - 100} = \frac{.1x^2 + 11x + 7000 - (.1(100)^2 + 11(100) + 7000)}{x - 100}$$

$$= \frac{.1x^2 + 11x - .1(100)^2 - 11(100)}{x - 100} = \frac{.1(x^2 - 100^2) + 11(x - 100)}{x - 100}$$

$$= \frac{.1(x-100)(x+100) + 11(x-100)}{x - 100}$$

$$= \frac{(x-100)(.1(x+100) + 11)}{x - 100}$$

$$= .1(x+100) + 11 \xrightarrow{x \to 100} .1(200) + 11 = 20 + 11 = \boxed{31 \frac{\$}{\text{unit}} = C'(100)}$$

$(x \neq 100)$

(c) 5pts **Since the average rate and the instantaneous rate are 31.1 \$/unit and 31 \$/unit, respectively, it would seem that *C* ' (*x*) is a reasonable approximation for marginal cost, and it's not unusual to see it be *the definition* of marginal cost.**

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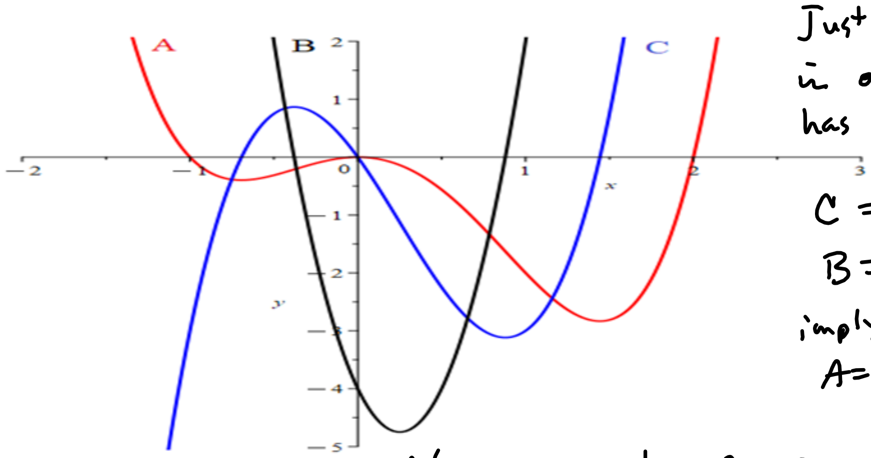
(4) (5pts) The graphs of A, B, & C are the graphs of f, f', and f'', not necessarily in that order



Just looking for zeros in one, where the other has a max/min...

$C = A'$

$B = C'$

implying

$A = f$, $C = f'$, $B = f''$

Continuing $A' < 0$ is when $C < 0$

$A' > 0$ " " $C > 0$ ✓

$C' > 0$ " " $B > 0$ ✓

$C' < 0$ " " $B < 0$

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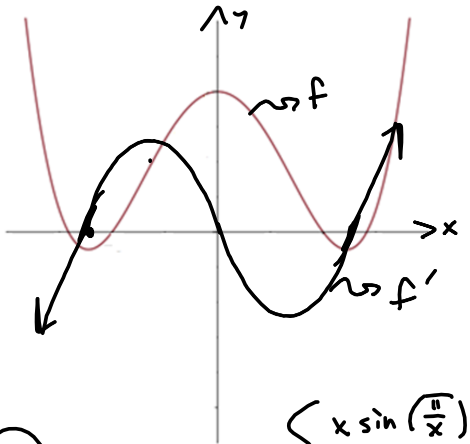
(5) (5pts) We sketch f' based on the graph of, below.



(6) (5pts) Let $g(x) = \begin{cases} x\sin\left(\frac{\pi}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Then $g'(0) \nexists$.

Proof $\lim_{h \to 0} \frac{g(0+h) - g(0)}{h} = \lim_{h \to 0} \frac{h\sin\left(\frac{\pi}{h}\right) - 0}{h} =$

$= \lim_{h \to 0} \sin\left(\frac{\pi}{h}\right) \nexists$, by previous work.

$\sin\left(\frac{\pi}{x}\right)$ oscillates between $y=1$ & $y=-1$ infinitely many times on <u>any</u> interval containing $x=0$.