2410 **Week 3 Written Assignment Solutions** H. Mills

1 (5pts) We're making a cube-shaped water tank. We want it to weigh 30Kg, which is 30,000 cm^3. This means each side of the cube is $\sqrt[3]{30000}$ cm in length and width.
We want the volume to be within 10 cm^3 of the desired 30,000 cm^3. How close to $\sqrt[3]{30000}$ must the length and width of each side of the cube be to guarantee this? We round to 5 places.

Let x = the side dimensions of the cube (in cm)

Then volume $= V(x) = x^3$ cm^3.

We want Volume within ± 10 cm^3 of 30000 cm^3.

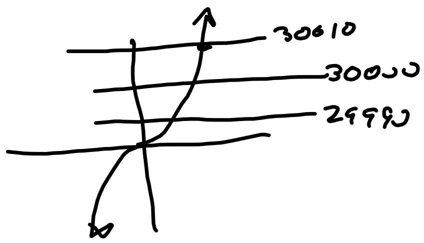
That means we want

$$|x^3 - 30000| < 10, \text{ i.e.}$$

$$-10 < x^3 - 30000 < 10 \longrightarrow$$

$$29990 < x^3 < 30010$$

$$\Longrightarrow \sqrt[3]{29990} < x < \sqrt[3]{30010}$$



$\delta_1 = \sqrt[3]{30000} - \sqrt[3]{29990} \approx$ 0.00345286424217

$\delta_2 = \sqrt[3]{30010} - \sqrt[3]{30000} \approx$ 0.00345209702424

Note $\delta_1 > \delta_2$. We want the smaller, δ_2, but both round down to $\boxed{0.00345 \text{ cm}}$

2410 W#3 MILLS

2 5pts $\lim_{x\to 3}(5x+2)=17$.

Proof

Let $\varepsilon>0$ be given. Define $\delta=\frac{\varepsilon}{5}$. Then $0<|x-3|<\delta \Longrightarrow$

$$|5x+2-17| = |5x-15| = 5|x-3| < 5\delta = 5\left(\frac{\varepsilon}{5}\right)=\varepsilon \; \blacksquare$$

3 5pts Let $f(x)=x^2+5x-7$. Then $\lim_{x\to 3} f(x)=17$.

Scratch: $|f(x)-L| = |x^2+5x-7-17| = |x^2+5x-24|$

$= |x-3||x+8| < \delta|x+8|$. Need bound on $|x+8|$

Assume $\delta \le 1$.

Then $0<|x-3|<1$

$\Longrightarrow -1 < x-3 < 1$

$\Longrightarrow 2 < x < 4$

$\Longrightarrow 2+8=10 < x+8 < 4+8=12$

$\Longrightarrow |x+8| < 12$ (b/c $|12|>|8|$)

Proof

Let $\varepsilon>0$ be given. Define $\delta=\min\{1,\frac{\varepsilon}{12}\}$.

Then $0<|x-3|<\delta \Longrightarrow$

$$|f(x)-L| = |x^2+5x-7-17| = |x^2+5x-24| = |x+8||x-3|$$

$$< |x+8|\delta < 12\delta \le 12\left(\frac{\varepsilon}{12}\right)=\varepsilon \; \blacksquare$$

2410 W #3 MILLS

(4) $f(x) = \begin{cases} x^2-4x & \text{if } x<3 \\ -2x+4 & \text{if } x \geq 3 \end{cases}$ is cnt^s on $(-\infty, 3)$ and $(3, \infty)$, separately

because each piece is a polynomial and hence cnt^s. We're not sure about $\lim_{x \to 3} f(x)$:

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (x^2-4x) = 3^2-4(3) = 9-12 = -3$.

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (-2x+4) = -2(3)+4 = -6+4 = -2$

$-2 \neq -3 \Longrightarrow \lim_{x \to 3} f(x) \nexists$, let alone agree with $f(3)$.

$\therefore$ f is NOT a cnt^s func.

(Jump discontinuity @ $x=3$)

2410 W#3 MILLS

5 (5pts) $g(x) = \begin{cases} x\sin\left(\frac{\pi}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is cnt's @ $x = 0$

Proof
we show that $\lim_{x\to 0} f(x) = f(0) = 0$:

We know that $-1 \leq \sin\left(\frac{\pi}{x}\right) \leq 1 \quad \forall\, x \neq 0$

If $x > 0$, then $-x \leq x\sin\left(\frac{\pi}{x}\right) \leq x$

and $\lim_{x\to 0}(-x) = \lim_{x\to 0}(x) = 0 \Longrightarrow$

$\lim_{x\to 0}\left(x\sin\left(\frac{\pi}{x}\right)\right) = 0$, by the Squeeze Theorem.

Likewise, $x < 0 \Longrightarrow$

$-x \geq x\sin\left(\frac{\pi}{x}\right) \geq x$

& once again $\lim_{x\to 0} x\sin\left(\frac{\pi}{x}\right)$ is squeezed between

$\lim_{x\to 0}(x)$ & $\lim_{x\to 0}(-x)$ & we are done. ∎