MAT 2410 70 Points 1.4 – The Tangent and Velocity Problems 1.5 – The Limit of a Function 1.6 – Calculating Limits Using Limit Laws

Be sure to follow <u>College Algebra formatting guidelines</u> in your work. They're the same for us as they are for College Algebra, except we're "2410" and not "1340," so "2410" in the top left corner, not "1340."

1. (5 pts) If a ball is thrown vertically at a speed of 50 miles per hour from a height of 10 feet, its height above the ground is given by $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$, where t is time in seconds, s is height in feet, and g is acceleration due to gravity, which in the English system is 32. This gives $s(t) = -16t^2 + 50t + 10$.

Find the average velocity of the ball (in ft/sec) for the time interval beginning at t = 1, and lasting for.

- i. ... 0.5 seconds
- ii. ... 0.1 seconds
- iii. ... 0.001 seconds

Round your answers to 4 decimal places.

- 2. Explain the meaning of the following, in words:
 - a. (5 pts) $\lim_{x \to 5} (f(x)) = 7$
 - b. (5 pts) $\lim_{x\to 5^-} (f(x)) = 7$ and $\lim_{x\to 5^+} (f(x)) = 6$. What does this say about f(5)?

Bonus – (2 pts) What do the two limits in part b say about the function at x = 5?

- c. (5 pts) Suppose that $\lim_{x \to 5} (f(x)) = \infty$. Provide a quick graphic that illustrates what's going on near x = 5.
- d. (5 pts) Suppose that $\lim_{x\to 5^-} (f(x)) = \infty$ and $\lim_{x\to 5^+} (f(x)) = -\infty$. Provide a quick graphic that illustrates what's going on near x = 5.
- 3. (5 pts) Consider the graph of a function *f* that's given on the right: Give the value of each of the following. If it doesn't exist, write "DNE" or use my short hand for "Does not exist."
 - a. $\lim_{x \to 3^{-}} (f(x))$
 - b. $\lim_{x \to 3^+} (f(x))$
 - c. $\lim_{x\to 3} (f(x))$



d. f(3)

- 4. (5 pts) Sketch the graph of a function that satisfies all of the conditions: $\lim_{x \to 3^{-}} (f(x)) = 4, \lim_{x \to 3^{+}} (f(x)) = 5, \lim_{x \to 6^{-}} (f(x)) = \infty, \lim_{x \to 6^{+}} (f(x)) = 7, \text{ with } f(3) = 4 \text{ and } f(6) = 11.$
- 5. Use the limit laws to evaluate the following limits, if possible. If not possible, say so, and explain why.

a. (5 pts)
$$\lim_{x \to 3} \left(\frac{x^2 + x - 6}{x^2 - 14x + 45} \right)$$

b. (5 pts)
$$\lim_{x \to 5} \left(\frac{x^2 + x - 6}{x^2 - 14x + 45} \right)$$

c. (5 pts)
$$\lim_{x \to 2} \left(\frac{x^2 + x - 6}{x^2 - 11x + 18} \right)$$

6. Let $f(x) = x^2 + x - 6$. Compute the limit of the following difference quotients:

a. (5 pts)
$$\lim_{h \to 0} \left(\frac{f(3+h) - f(3)}{h} \right)$$

b. (5 pts) $\lim_{x \to 3} \left(\frac{f(x) - f(3)}{x - 3} \right)$
c. (5 pts) $\lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$
d. (5 pts) $\lim_{x \to c} \left(\frac{f(x) - f(c)}{x - c} \right)$