2410 WEEK2 SOLNS H. MILLS

(1) Let $s$ = the height of a ball in free fall as a function of $t$ = .. number of seconds after launch. Then

$s = -\frac{1}{2}gt^2 + v_0 t + s_0$, where

$g = 32 \frac{ft}{s^2}$ is acceleration due to gravity (Downward),

$v_0$ = initial velocity (how fast it was thrown upward) in $\frac{ft}{sec}$

$s_0$ = .. height in feet.

In this problem, $v_0 = 50 \frac{ft}{s}$ & $s_0 = 10$, so

$$s(t) = -16t^2 + 50t + 10$$

We find the average velocity of the ball for the time interval starting at $t=1$ and lasting ...

(a) ... 0.5 sec

(b) ... 0.1 sec

(c) ... .001 sec

What this is saying is "Compute $m_{AVG} = \frac{s(1+h) - s(1)}{1+h - 1}$ for

$h = 0.5, 0.1, \& \; 0.001$

$$s(1) = -16(1)^2 + 50(1) + 10 = -16 + 60 = 44 \text{ ft}$$

The way I did this on Desmos was define $g(h)$ to be the difference quotient for $s$ on $[1, 1+h]$:

$$g(h) \quad \frac{s(1+h) - s(1)}{1+h-1} = \frac{s(1+h) - s(1)}{h}$$

$$= \frac{-16(1+h)^2 + 50(1+h) + 10 - 44}{h}$$

(a) $g(.5) = 10$

(b) $g(.1) = 16.4$

(c) $g(.001) = 17.984$

$\left( g(.00001) = 17.99998400 \right)$

**These happen to be exact. Usually, when you go to a decimal, it's understood to be an approximation, and you use the wavy "=" relational operator.**

2410

H. MILLS

**Desmos Work:**

1. $s(t) = -16t^2 + 50t + 10$
2. $s(1)$ $= 44$
3. $s(1.5)$ $= 49$
4. $\frac{(s(1.5) - s(1))}{1.5 - 1}$ $= 10$
5. $\frac{(s(1.5) - 44)}{1.5 - 1}$ $= 10$
6. $q(h) = \frac{(-16(1+h)^2 + 50(1+h) + 10 - 4}{h}$
7. $q(.1)$
8. $q(.5)$ $= 10$
9. $q(.001)$ $= 17.984$

2410 MILLS

(a) $\lim_{x \to 5} f(x) = 7$ means I can make $f(x)$ as close to $y=7$ as I want, by taking $x$ sufficiently close to $x=5$.

It reads as "Limit as 'x' approaches '5' of f(x) is 7."

(b) $\lim_{x \to 5^-} f(x) = 7$ and $\lim_{x \to 5^+} f(x) = 6$ means

f(x) approaches 7 as x approachs 5 from the left, and

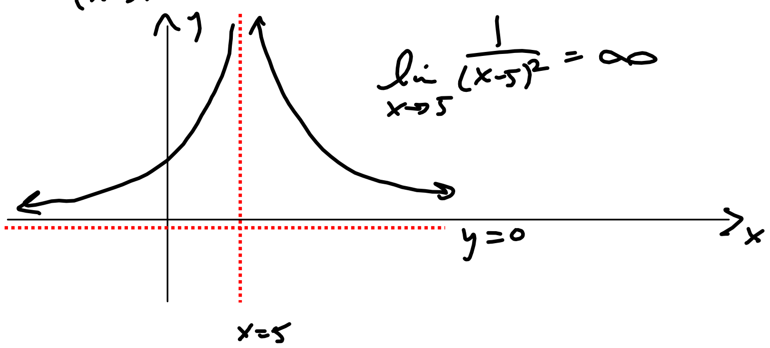
" " 6 " " " " " " right.

Bonus 2pts This says nothing about f(5).

2410 Hi Mills

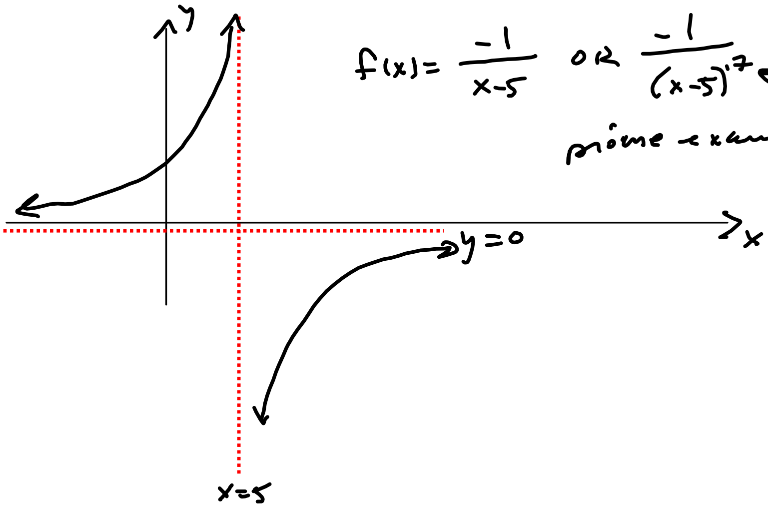
(c) $\lim_{x \to 5} f(x) = \infty$ means $f(x)$ grows arbitrarily large as $x$ comes arbitrarily close to $x=5$, without touching

$\frac{1}{(x-5)^2}$ is a prime example




$\lim_{x \to 5} \frac{1}{(x-5)^2} = \infty$

(d) $\lim_{x \to 5^-} f(x) = \infty$ & $\lim_{x \to 5^+} f(x) = -\infty$




$f(x) = \frac{-1}{x-5}$ or $\frac{-1}{(x-5)^7}$ is a prime example

Any odd #

2410

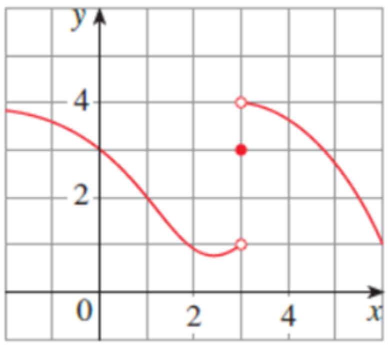
(3) The graph on the right belongs to $f(x)$.

(a) $\lim_{x \to 3^-} f(x) = 1$

(b) $\lim_{x \to 3^+} f(x) = 4$

(c) $\lim_{x \to 3} f(x) \nexists$, because $1 \neq 4$!

(d) $f(3) = 3$

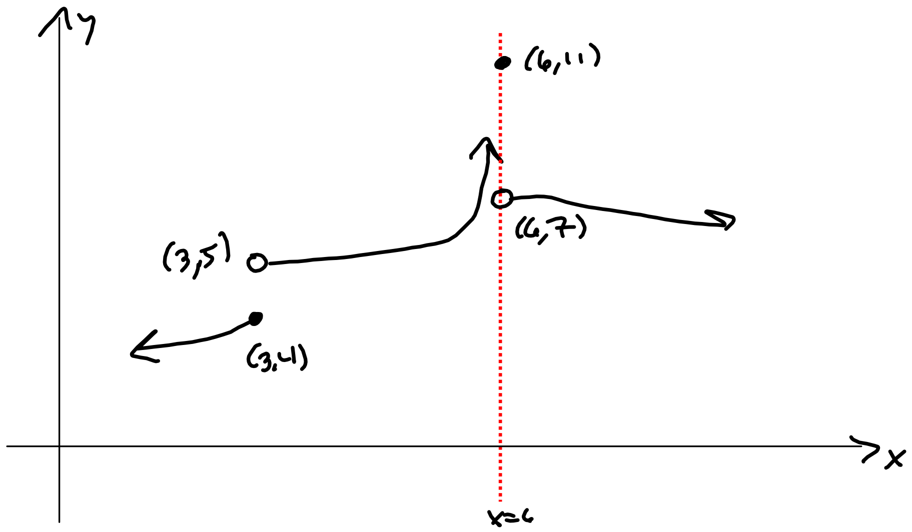


2.4.10

H. MILLS

(4) We sketch a plausible graph of a function satisfying the following:

$\lim_{x\to 3^-} f(x)=4$, $\lim_{x\to 3^+} f(x)=5$, $\lim_{x\to 6^-} f(x)=\infty$, $f(3)=4$, $f(6)=11$



2410

H. MILLS

5 a) Let $f(x) = \frac{x^2+x-6}{x^2-14x+45} = \frac{(x+3)(x-2)}{(x-9)(x-5)}$

$\longrightarrow \lim_{x\to 3} f(x) = \frac{(3+3)(3-2)}{(3-9)(3-5)} = \frac{(6)(1)}{(-6)(-2)} = \frac{6}{12} = \boxed{\frac{1}{2} = \lim_{x\to 3} f(x)}$

$\lim_{x\to c} f(x) = f(c)$ on $f$'s domain, as it is a rational function.

b) $\lim_{x\to 5} f(x) = \lim_{x\to 5} \frac{(x+3)(x-2)}{(x-9)(x-5)}$ $\boxed{\nexists}$, b/c $x-5$ in denominator (and nothing cancels)

c) $g(x) = \frac{x^2+x-6}{x^2-11x+18} = \frac{(x+3)(x-2)}{(x-9)(x-2)} = \frac{x+3}{x-9}$ $(x \neq 2)$ $\xrightarrow{x\to 2}$ $\boxed{\frac{5}{-7} = \lim_{x\to 2} g(x)}$

2410 H. MILLS

6. Let $f(x) = x^2+x-6$.

a) Then $\frac{f(3+h)-f(3)}{h} = \frac{(3+h)^2+(3+h)-6-(3^2+3-6)}{h}$

$= \frac{3^2+2\cdot 3h+h^2+3+h-6-6}{h} = \frac{6h+h^2+h}{h} = \frac{7h+h^2}{h}$

$= \frac{7+h}{1} \xrightarrow{h\to 0} \boxed{7 = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}}$
$(h\neq 0)$

b) $\frac{f(x)-f(3)}{x-3} = \frac{x^2+x-6-6}{x-3} = \frac{x^2+x-12}{x-3} = \frac{(x+4)(x-3)}{x-3}$

$= x+4 \xrightarrow{x\to 3} 3+4 = \boxed{7 = \lim_{x\to 3} \frac{f(x)-f(3)}{x-3}}$
$(x\neq 3)$

c) $\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+(x+h)-6-(x^2+x-6)}{h}$

$= \frac{x^2+2xh+h^2+x+h-6-x^2-x+6}{h} = \frac{2xh+h^2+h}{h}$

$= 2x+h+1 \xrightarrow{h\to 0} \boxed{2x+1 = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}$
$(h\neq 0)$

d) $\frac{f(x)-f(c)}{x-c} = \frac{x^2+x-6-(c^2+c-6)}{x-c}$

$= \frac{x^2+x-6-c^2-c+6}{x-c} = \frac{x^2+x-c^2-c}{x-c}$

$= \frac{x^2-c^2+x-c}{(x-c)} = \frac{(x-c)(x+c)+1(x-c)}{(x-c)} = \frac{(x-c)(x+c+1)}{x-c}$

$= x+c+1 \xrightarrow{x\to c} \boxed{2c+1 = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}}$
$x\neq c$