

$$\textcircled{a} f(0) = 1$$

$$\textcircled{b} g(6) = 2$$

$$\textcircled{c} g(4) > f(4)$$

$g(4)$  is larger than  $f(4)$

$$\textcircled{d} g(6) = 2$$

$$\textcircled{e} f(x) < g(x) \text{ on } (-3, 5)$$

$$\textcircled{f} g \text{ is increasing on } [-4, 1]$$

$$\textcircled{g} g \text{ is decreasing on } [1, 7]$$

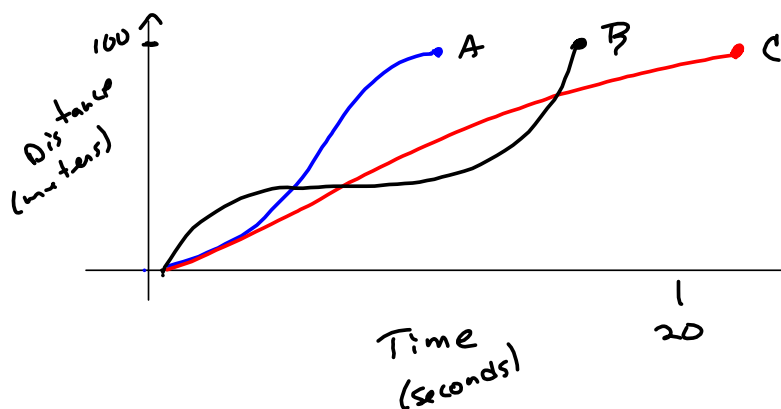
$$\textcircled{h} \mathcal{D}(f) = \text{Domain of } f = \{x \mid f(x) \text{ is defined}\}$$

$$= [-4, 6] = \mathcal{D}(f)$$

$$\mathcal{R}(f) = [1, 8.3] = \text{Range of } f = \{y \mid y = f(x) \text{ for some } x \in \mathcal{D}(f)\}$$

$$\textcircled{i} \mathcal{D}(g) = [-6, 7] \text{ and } \mathcal{R}(f) = [-2, 8.3]. \text{ (ish)}$$

- ② The graph describes distance as a function of time.



Runner A ran a steady race, and finished 1<sup>st</sup>.

Runner B was fastest of the blocks, but must have fallen, b/c he stopped moving for a while, finishing 2<sup>nd</sup>.  
 Runner C was slow, and finished last, in over 20 seconds.

③  $f(x) = x^2 - 3x + 2 \implies$

①  $\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 - 3(2+h) + 2 - (2^2 - 3(2) + 2)}{h}$

$$= \frac{4 + 4h + h^2 - 6 - 3h + 2 - 4 + 6 - 2}{h}$$

$$= \frac{4h + h^2 - 3h}{h} = \frac{h + h^2}{h} = 1 + h \xrightarrow{h \rightarrow 0} 1$$

( $h \neq 0$ )

②  $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 3(x+h) + 2 - (x^2 - 3x + 2)}{h}$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2}{h} = \frac{2xh + h^2 - 3h}{h}$$

$$= \frac{2x + h - 3}{h} \xrightarrow{h \rightarrow 0} \boxed{2x - 3 = f'(x)}$$

( $h \neq 0$ )

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(4)  $f(x) = \frac{x-3}{x^3-27} = \frac{x-3}{(x-3)(x^2+3x+9)} = \frac{1}{x^2+3x+9}$   
 (x ≠ 3)

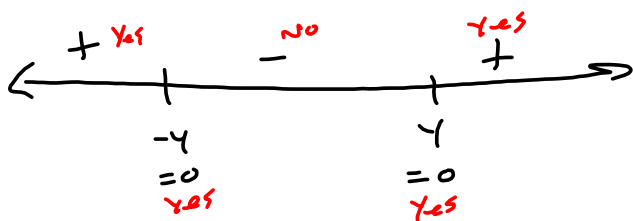
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$D(f) = \mathbb{R} \setminus \{3\}$

$x^3-27$  has root  $\sqrt[3]{27} = 3$

(5)  $f(x) = \sqrt{x^2-16} \rightarrow$

$D(f) = \{x \mid x^2-16 \geq 0\} = \{x \mid (x-4)(x+4) \geq 0\}$



want  $\geq 0$ , i.e., "+" or "= 0"

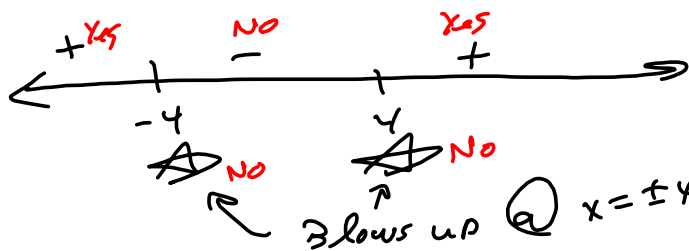
Sign pattern for  $x^2-16$

$\Rightarrow D(f) = (-\infty, -4] \cup [4, \infty)$

(6)  $g(x) = \frac{1}{\sqrt{x^2-16}} \rightarrow D(g)$  is same as  $D(f)$ , almost.

Not quite, b/c we also need  $x^2-16 \neq 0$ , i.e.,

$x^2-16 > 0 =$

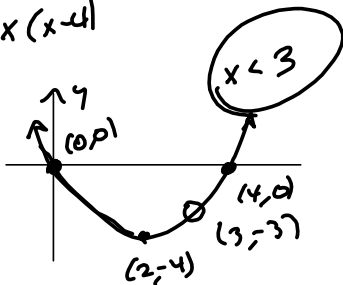


$D(g) = (-\infty, -4) \cup (4, \infty)$

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 (6) we sketch  $f(x) = \begin{cases} x^2 - 4x & \text{if } x < 3 \\ -2x + 3 & \text{if } x \geq 3 \end{cases}$

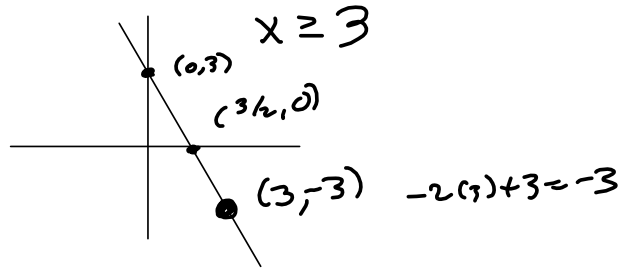
$x^2 - 4x = x(x - 4)$



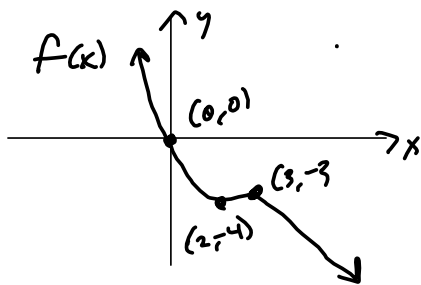
$f(2) = 2^2 - 4(2) = 4 - 8 = -4$   
 $f(3) = 3^2 - 4(3) = 9 - 12 = -3$

$y = -2x + 3$

x	y
0	3
3/2	0



Cool! they connect @ the suture point  $x = 3$ !



⑦ Find a linear equation that models temperature  $T$  (in degrees Fahrenheit), as a function of  $n$  = the # of times a cricket chirps per minute if

$$(n_1, T_1) = (115, 72) \text{ \& } (n_2, T_2) = (180, 81) \text{ i.e.,}$$

$$115 \text{ chirps/min} \rightsquigarrow 72^\circ \text{ \& }$$

$$180 \text{ chirps/min} \rightsquigarrow 81^\circ$$

Lines are easier w/  $x$ 's \&  $y$ 's:

$$y = T$$

$$x = n$$

$$(x_1, y_1) = (n_1, T_1) = (115, 72)$$

$$(x_2, y_2) = (n_2, T_2) = (180, 81)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{81 - 72}{180 - 115} = \frac{9}{65} = m \rightarrow$$

$$y = m(x - x_1) + y_1$$

$$y = \frac{9}{65}(x - 115) + 72, \text{ i.e.,}$$

$$\boxed{T = \frac{9}{65}(n - 115) + 72}$$

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⑧ To graph  $-3f(-2x-14)+11$  from  $f(x)$ ,  
I'd factor out the "-2" inside, and get  
 $-3f(-2(x+7))+11$  and I would i.i.

∴ Flip vertically and stretch vertically by a factor  
of -3, i.e.,  $y \mapsto -3y$ ,  $(-3f(x))$

Then I'd shrink towards  $y$ -axis by a factor of 2  
and flip about the  $y$ -axis  $(-3f(-2x))$ .  $x \mapsto -\frac{1}{2}x$

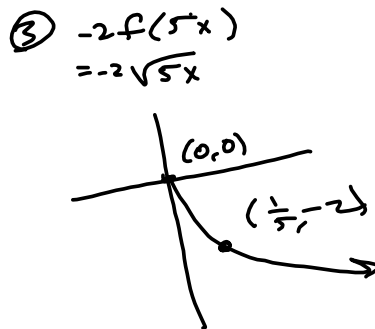
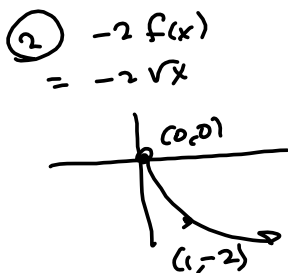
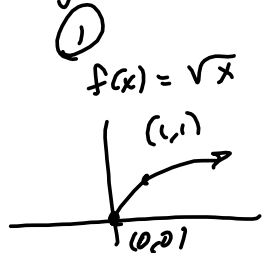
Then I'd shift left 7 ∴  $(-3f(-2(x+7)))$ .  $x \mapsto x-7$

Finally, I'd move it all up 11 units.  $y \mapsto y+11$

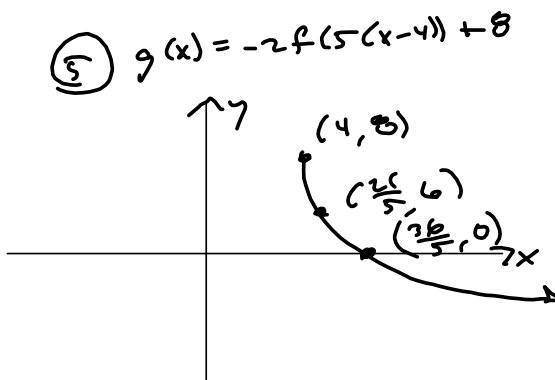
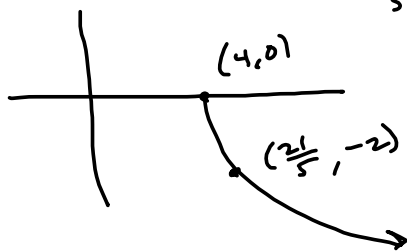
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⑨  $g(x) = -2\sqrt{5x-20} + 8 = -2\sqrt{5(x-4)} + 8$



④  $-2f(5(x-4)) = -2\sqrt{5(x-4)}$   
 $x \mapsto x+4$   
 $\frac{1}{5} + 4 = \frac{1+20}{5} = \frac{21}{5}$



x-int:  
 $g(x) = 0 \rightarrow$   
 $-2\sqrt{5x-20} + 8 = 0$   
 $-2\sqrt{5x-20} = -8$   
 $\sqrt{5x-20} = 4$   
 $5x-20 = 4^2 = 16$   
 $5x = 36$   
 $x = \frac{36}{5} \rightarrow (\frac{36}{5}, 0)$

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$$\textcircled{10} \quad f(x) = x^2 - 2x - 3 \quad \& \quad g(x) = 2x^2 - 3x - 5 \quad \rightarrow$$

$$\textcircled{a} \quad (f+g)(x) = f(x) + g(x) = x^2 - 2x - 3 + 2x^2 - 3x - 5$$

$$= \boxed{3x^2 - 5x - 8 = f(x) + g(x)} \quad \left| \quad \mathcal{D}(f+g) = \mathbb{R} \right.$$

$$\textcircled{b} \quad R(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 2x - 3}{2x^2 - 3x - 5} = \frac{(x-3)(x+1)}{(2x-5)(x+1)} = \frac{x-3}{2x-5} \quad (x \neq -1)$$

$$\mathcal{D}(R) = \mathbb{R} \setminus \left\{ \frac{5}{2}, -1 \right\} \quad \text{OR} \quad (-\infty, -1) \cup \left(-1, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$$

$$\text{OR} \quad \left\{ x \mid x \neq \frac{5}{2} \text{ and } x \neq -1 \right\} =$$

$$= \left\{ x \mid x \neq \frac{5}{2} \text{ OR } -1 \right\} = \left\{ x \mid x \notin \left\{ -1, \frac{5}{2} \right\} \right\}$$

$$= \left\{ x \mid x \notin \left\{ \frac{5}{2}, -1 \right\} \right\}$$