MAT 2410-R11 Written Midterm, Spring, 2025 (5 pts) Name ________________
150 Points Covers Chapters 1 – 3

Do all your work and circle all your final answers on the blank paper provided. Do your own work.

You don't need to write out the questions, but you do need to write out your answers as completely as possible.

Spend no more than 5 minutes on any one problem before moving on to the next. If you get stuck, start the next problem on a fresh sheet of paper.

Be sure to submit all the problems in the order in which they appear on the test, itself. If you don't, that's 10% off the top.

Remember that partial credit is awarded liberally. Final answers are important, but most of the points are in the supporting work.

1. Evaluate the following limits, if they exist. If one does not exist, explain why.
   a. (5 pts) $\lim_{x\to 4^-}\dfrac{x^2+4x-32}{|x-4|}$
   b. (5 pts) $\lim_{x\to 4^+}\dfrac{x^2+4x-32}{|x-4|}$
   c. (5 pts) $\lim_{x\to 4}\dfrac{x^2+4x-32}{|x-4|}$

2. Consider the piecewise-defined function $f(x)=\begin{cases} x^2-5x-14 & \text{if } x<3 \\ 3x-29 & \text{if } x\ge 3 \end{cases}$.
   a. (5 pts) Sketch the graph of $f(x)$. Label the x- and y-intercepts, the suture point(s), and the vertex of the quadratic piece, if it's in the picture. When I say "Label," I mean an ordered pair, like (0, 5), next to the point.
   b. (5 pts) On what interval(s) is $f(x)$ continuous? Explain.
   c. (5 pts) On what interval(s) is $f(x)$ differentiable? Explain.

3. (5 pts) Simplify the limit of the difference quotient $\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ for $f(x)=x^2+2x-10$.

4. The point $P(1,-7)$ lies on the graph of $f(x)=x^2+2x-10$.
   a. (5 pts) Write the equation of the tangent line $L_1(x)$ to $f(x)$ at $x=1$.
   b. (5 pts) Sketch a graph of $f(x)$ and $L_1(x)$ on the same set of coordinate axes.

5. (5 pts) Prove that $\lim_{x\to -2}(5x+7)=-3$, using the $\varepsilon-\delta$ definition of limit.

6. (5 pts) Prove that the equation $f(x) = x^4 - 4x^3 + 6x^2 + 28x - 91$ has a root in the interval $(0,5)$, but *do not solve*!

7. Differentiate the following with respect to the indicated independent variable. **Do not simplify!**

   a. (5 pts) $f(x) = \sqrt[3]{x^5} - 27x^{\frac{7}{4}} + -\frac{11}{x^6}$; x.

   b. (5 pts) $g(x) = \cos(5x)\sec(3x)$; x.

   c. (5 pts) $h(\beta) = \dfrac{12\beta^3 + 2\beta}{\tan(\beta)}$; β.

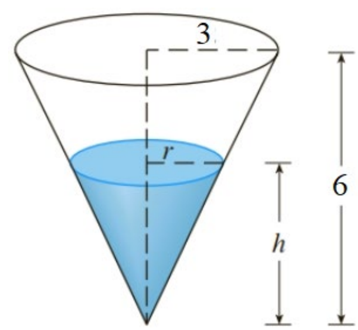
   d. (5 pts) $r(w) = (w^2 + 11w + 5)^4 (2w + 6)^3$; w.

8. Consider the relation $x^2 - 3xy + 4y^2 = \cos(y)$.

   a. (5 pts) Use implicit differentiation to find $y' = \dfrac{dy}{dx}$

   b. (5 pts) Find an equation of the tangent line to the curve at the point $(1,0)$.

9. (10 pts) A water tank has the shape of an inverted circular cone with base radius 3 m and height 4 m. If water is being pumped out of the tank at a rate of 3 $\dfrac{\text{m}^3}{\text{min}}$, find the rate at which the water level is dropping when the water is 4 m deep. Hints: The volume of a circular cone is $V = \dfrac{1}{3}\pi r^2 h$. It is possible (and essential) to write V as a function of r for this situation.



10. (10 pts) If the minute hand of a clock has length r (in centimeters), find the rate at which it sweeps out area as a function of r.

11. Let $R(x) = \dfrac{(x-2)(x-5)}{x-8} = \dfrac{x^2 - 7x + 10}{x-8}$

   a. (5 pts) What is the domain of R? Give any vertical asymptotes for R.

   b. (5 pts) Find the slant asymptote for R.

   c. (5 pts) Find $R'(x)$. Find its zeros and any points where it blows up.

   d. (5 pts) Create a sign pattern for $R'(x)$.

   e. (5 pts) Find $R''(x)$.

f. (5 pts) Find the zeros of $R''(x)$ and any points where it blows up.

g. (5 pts) Create a sign pattern for $R''(x)$.

h. (5 pts) Sketch a graph of the Asymptotes of R.

i. (**Bonus 10 pts**) Sketch a complete graph of R. If you encounter a max/min or inflection point where the x-value is something like $x = 7 - 5\sqrt{3}$, it will suffice to write $B = \left(7 - 5\sqrt{3}, R\left(7 - 5\sqrt{3}\right)\right)$ in a list of points, A, B, C, etc., off to the side, with the points on the graph labeled A, B, C, etc., as long as the points are generally in the right location. I don't want you to have to evaluate messy functions with messy inputs, here.

BONUS:

1. (5 pts) Prove that $\lim_{x \to 2}\left(3x^2 - 2x - 5\right) = 3$, using the $\varepsilon - \delta$ definition of limit.

2. (5 pts) Compute the derivative of $f(x) = x^{\frac{4}{3}}$ by the limit definition of the derivative. Hint: Use this formulation: $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ and be mindful of special products.

3. (5 pts) Find the x-intercept of the tangent line $L_{x_1}(x)$ to a differentiable function $f(x)$ at x_1

4. (5 pts) If $x_1 = \frac{3\pi}{4}$ and $f(x) = 2\sin(3x) + 5\sin(2x)$, what does Newton's Method say x_2 is?

5. (5 pts) See if you can *squeeze* out a *convincing* argument to support the statement

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous on $(-\infty, \infty)$.

6. (5 pts) Let $f(x) = x^3 - 6x^2 + 5x - 4$ Find $c \in (0,4)$ such that $f'(c)$ equals the average slope of f over the interval $[0,4]$. How did you know such a c existed before you started?

7. (5 pts) Suppose $f''(x) = 20x^3 - 12x^2 + 6x - 4$. Given $f(1) = f'(1) = 5$, what is $f(x)$

Note to the Proctor: Please scan the written pages and the front page of this test (with their name on it).

Please make sure that the student turns in their cheat sheet and all paper with any writing on it.