MAT 2410 Written Final Name_____
150 Points Concentration on Chapters 4 – 6

Do your own work. Circle Final Answers. Problems go UNDER one another, not side-by-side. The more cramped your work is, the less partial credit I will be able to – let alone be inclined to – award.

1. Let $f(x) = x^2 - 5x$.

   a. (5 pts) Use the limit definition of the derivative to find $f'(x)$

   b. (5 pts) Use the limit definition of the definite integral to evaluate $\int_{-1}^{1} f(x)\,dx$.

2. Consider the region bounded by $y = \sin(x)$, $y = 1$, $x = 0$, and $x = \frac{\pi}{2}$. in two ways.

   a. (5 pts) Sketch the region.

   b. (5 pts) Write the area of the region as an integral with respect to *x*. Draw a representative rectangle on the sketch from part a.

   c. (5 pts) Evaluate the integral from part b.

   d. (5 pts) Sketch the region, again.

   e. (5 pts) Write the area as an integral with respect to *y*. Draw a representative rectangle on the sketch from #2*d*. Do not evaluate the integral.

   f. (**Bonus** 5 pts) Evaluate your answer to #2*e*. Points:Time = small.

   g. (5 pts) Suppose we rotated the region about the line $y = -1$. Sketch the graph of the solid of revolution obtained. Include a representative disk or washer.

   h. (5 pts) Write the integral representing the volume of the solid of revolution in #2*g*.

   i. (**Bonus** 5 pts) Evaluate the integral you wrote for #2*h*.

   j. (**Bonus** 5 pts) Write the integral for revolving the region about the line $x = -1$. Not fun.

   k. (**Bonus** 5 pts) Evaluate the integral in #2*j*. (Beyond the scope of the course)

3. We explore absolute value. Let $f(x) = 2\sin(x) - \sqrt{3}$

   a. (5 pts) Sketch a complete graph of $f(x)$ on the interval $[0, 2\pi]$. That means all intercepts, local extreme points and inflection points. This requires no calculus, if you're familiar with the graph of sine.

   b. (5 pts) Evaluate $\int_{0}^{2\pi} f(x)\,dx$.

c. (5 pts) Sketch a complete graph of $\left|f\left(x\right)\right|$ on the interval $\left[0,2\pi\right]$. It doesn't require calculus to find local extreme points and inflection points, if you know the graph of sine.

d. (5 pts) Evaluate $\int_0^{2\pi}\left|f\left(x\right)\right|dx$.

4. Evaluate the indefinite integrals. **Do Not Simplify**.

a. (5 pts) $\int\frac{1}{\left(2x-7\right)^4}dx$

b. (5 pts) $\int x\left(2x-7\right)^{-5}\,dx$

c. (5 pts) $\int\sin^5\left(x\right)\cos^3\left(x\right)dx$

d. (5 pts) $\int 2^{3x}\,dx$

5. Let f be a smooth function such that $f'\left(x\right)=3x^2-2x$

a. (5 pts) What's the net change in f over the interval $\left[0,2\right]$?

b. (5 pts) What's the *total* change in f over the interval $\left[0,2\right]$?

6. Differentiate the following with respect to *x*. **Do not simplify**:

a. (5 pts) $3\cdot 7^{\sqrt{x}}$

b. (5 pts) $y=\ln\left(\frac{\tan^7\left(x\right)\sqrt{x^2+1}}{\sqrt[13]{\cos\left(5x\right)}}\right)$

c. (5 pts) $y=\log_7\left(x\sin\left(5x\right)\right)$

d. (5 pts) $y=\left[x^2+5x\right]^{\tan(x)}$

e. (5 pts) $y=\int_0^x\frac{\sin\left(5t\right)+t^3}{\sqrt{\cos^2\left(t^2\right)+1}}dt$

f. (5 pts) $y=\int_0^{2x^2-3x}\frac{\sin\left(5t\right)+t^3}{\sqrt{\cos^2\left(t^2\right)+1}}dt$

7. The function $f\left(x\right)=\sqrt{x+36}-3$ is 1-to-1 on its domain.

a. (5 pts) Find the domain and range of f.

b. (5 pts) Find the inverse function $f^{-1}(x)$. State its range and the restricted domain required for domains and ranges to match up.

c. (5 pts) Find $\left(f^{-1}\right)'(2)$, directly, by differentiating your answer for part b.

d. (5 pts) Find $\left(f^{-1}\right)'(2)$ by applying a theorem regarding derivatives of inverse functions.

8. (5 pts) Sand is being dumped from a conveyor belt at a rate of 30 $\frac{\text{ft}^3}{\text{min}}$. The resulting pile of sand forms the shape of a cone whose base radius and height are the same. How fast is the height of the pile increasing when the pile is 10 ft high? Recall, the volume of a cube of height *h* and radius *r* is $V = \frac{1}{3}\pi r^2 h$.

9. (5 pts) Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x\to 5}(2x-3) = 7$.

10. (5 pts) Find y' if $x^2 + y^2 - 2xy = \sin(\pi xy)$.

BONUS SECTION

1. (**Bonus** 5 pts) Find the equation of the tangent line to the graph of $x^2 + y^2 - 2xy = \sin(\pi xy)$ at the point $(1,1)$. This is a continuation of #10.

2. (**Bonus** 5 pts) Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x\to 5}(x^2 - 3) = 22$

3. (**Bonus** 5 pts) Use logarithmic differentiation to find $y' = \frac{dy}{dx}$ for $y = \frac{(x-3)(x+2)}{(x+5)(x-7)}$. Do not simplify!

4. (**Bonus** 5 pts) Sketch a complete graph of $R(x) = \frac{(x+3)(x-3)}{x-5}$