1a 5pts $f(x) = x^2 - 5x \longrightarrow$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5(x+h) - (x^2 - 5x)}{h} = \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$$

$$= \frac{2xh + h^2 - 5h}{h} = \frac{h(2x + h - 5)}{h} \xrightarrow{h \to 0} \boxed{2x - 5 = f'(x)}$$

b 5pts

$$I = \int_{-1}^{1} (x^2 - 5x)\,dx = \int_{-1}^{1} x^2\,dx - 5\int_{-1}^{1} x\,dx = \int_{-1}^{1} x^2\,dx$$

, because $y = -5x$ is odd and $[a,b] = [-1,1]$ is a symmetric interval.

$$\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{n} = \frac{2}{n}$$

$$x_k = a + k\left(\frac{b-a}{n}\right) = -1 + \frac{2}{n}k = \frac{2k}{n} - 1$$

$$f(x_k) = \left(\frac{2k}{n} - 1\right)^2 = \frac{4k^2}{n^2} - 2\left(\frac{2k}{n}\right) + 1 = \frac{4k^2}{n^2} - \frac{4k}{n} + 1$$

$$\Longrightarrow \Delta x \sum_{k=1}^{n} f(x_k) = \frac{2}{n} \sum_{k=1}^{n} \left(\frac{4k^2}{n^2} - \frac{4k}{n} + 1\right)$$

$$= \frac{2}{n}\left[\frac{4}{n^2}\sum_{k=1}^{n} k^2 - \frac{4}{n}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1\right]$$

$$= \frac{8}{n^3}\left(\frac{n^3 + \cdots}{3}\right) - \frac{8}{n^2}\left(\frac{n^2 + \cdots}{2}\right) + \frac{2}{n}(n)$$

$$\xrightarrow{n \to \infty} \frac{8}{3} - \frac{8}{2} + 2 = \frac{16 - 24 + 12}{6} = \frac{28 - 24}{6} = \frac{4}{6} = \boxed{\frac{2}{3} = I}$$

More clever

Using symmetry:

$$\int_{-1}^{1}(x^2-5x)\,dx = \int_{-1}^{1}x^2\,dx = 2\int_0^1 x^2\,dx$$

$a=0, b=1 \Rightarrow \Delta x = \frac{b-a}{n} = \frac{1}{n}$

$x_k = a+k\Delta x = 0+k\cdot\frac{1}{n} = \frac{k}{n}$

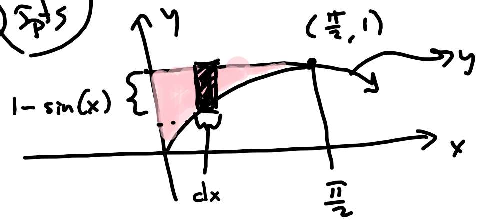
$f(x_k) = \left(\frac{k}{n}\right)^2 = \frac{k^2}{n^2}$

$$\Delta x\sum_{k=1}^{n} f(x_k) = \frac{1}{n}\sum_{k=1}^{n}\frac{k^2}{n^2} = \frac{1}{n^3}\sum_{k=1}^{n}k^2 = \frac{1}{n^3}\left[\frac{n^3+\cdots}{3}\right]$$

$$\xrightarrow{n\to\infty} \frac{1}{3} = \int_0^1 f(x)\,dx = \frac{1}{2}\int_{-1}^{1}f(x)\,dx \Rightarrow \int_{-1}^{1}f(x)\,dx = 2\left(\frac{1}{3}\right) = \boxed{\frac{2}{3}}$$

(2) $f(x)=\sin(x)$, $y=1$, $x=0$, $x=\frac{\pi}{2}$

(a) 5pts



$\rightarrow y=\sin(x)$ OR $x=\arcsin(y)$

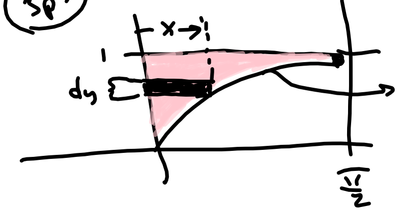
(b) 5pts upper $= y_u = 1$, $y_\ell = \sin(x) =$ lower

$$\Delta x \sum_{k=1}^{n} f(x_k) \xrightarrow{n\to\infty} \int_0^{\frac{\pi}{2}} (y_u - y_\ell)\,dx = \boxed{\text{Area} = \int_0^{\frac{\pi}{2}} (1-\sin(x))\,dx}$$

(c) 5pts

$$= \Big[\, x + \cos(x) \,\Big]_0^{\frac{\pi}{2}} = \frac{\pi}{2} + \cos\left(\frac{\pi}{2}\right) - \Big(0 + \cos(0)\Big) = \boxed{\frac{\pi}{2} - 1}$$

(d) 5pts Same area wrt "y"



$\rightarrow x = g(y) = \arcsin(y)$

(e) 5pts

$y = \sin(x) = f(x)$, $x \in [0, \frac{\pi}{2}]$

$\boxed{\arcsin(y) = x = g(y)}$, $y \in [0, 1]$

$$\int_0^1 x\,dy = \boxed{\int_0^1 \arcsin(y)\,dy = \text{Area}}$$

f) Bonus 5pts

$$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\cos(\arcsin(x))}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

Deriving $\int \arcsin(x)\,dx$ from what's given.

$$\int \arcsin(x)\,dx$$

$u = \arcsin(x)$ $\quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad v = x$

$$= uv - \int v\,du = x\arcsin(x) - \int x\left(\frac{1}{\sqrt{1-x^2}}\right)dx$$

$u = 1-x^2$

$du = -2x\,dx$

$$= x\arcsin(x) + \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}(-2x\,dx)$$

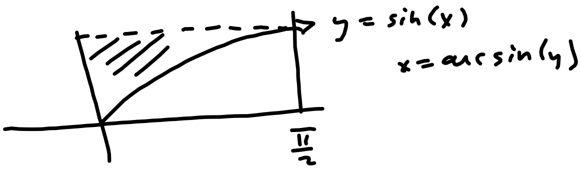
$$= x\arcsin(x) + \frac{1}{2}\left(\frac{2}{1}\right)(1-x^2)^{\frac{1}{2}} + C$$

$$= x\arcsin(x) + \sqrt{1-x^2} + C$$

$$\int_0^1 \arcsin(y)\,dy = \left[y\arcsin(y) + \sqrt{1-y^2}\right]_0^1$$

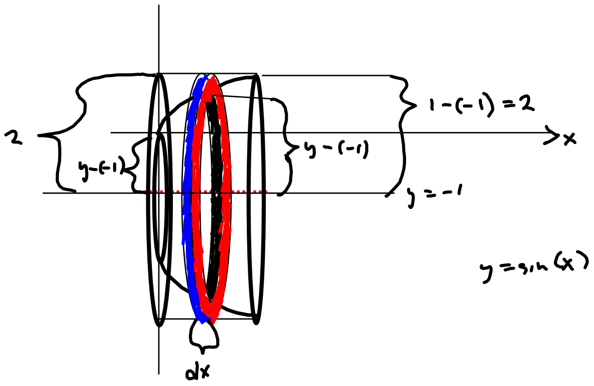
$$= 1\arcsin(1) + 0 - (0+1)$$

$$= \boxed{\frac{\pi}{2} - 1}$$



g) 5pts Rotate the region about the line $y=-1$

sketch



Stop!

h) 5pts $\pi\int_2^3 (y_o^2 - y_i^2)\,dx = \boxed{\pi\int_0^{\frac{\pi}{2}}\left(2^2 - (\sin(x)+1)^2\right)dx}$

i) 5pts Bonus

$$= \pi\int_0^{\frac{\pi}{2}}\left(4 - (\sin^2(x) + 2\sin(x) + 1)\right)dx$$

$$= \pi\int_0^{\frac{\pi}{2}}\left(4 - \sin^2(x) - 2\sin(x) - 1\right)dx = \pi\int_0^{\frac{\pi}{2}}\left(3 - \sin^2(x) - 2\sin(x)\right)dx$$

$$= \pi\left[3x\right]_0^{\frac{\pi}{2}} - \pi\int_0^{\frac{\pi}{2}}\left(\frac{1-\cos(2x)}{2}\right)dx - 2\pi\int_0^{\frac{\pi}{2}}\sin(x)\,dx$$

$$= \frac{\pi}{2}\left[\frac{3\pi}{2} - 0\right] - \frac{\pi}{2}\left[x - \frac{1}{2}\sin(2x)\right]_0^{\frac{\pi}{2}} + 2\pi\left[\cos\frac{\pi}{2} - \cos(0)\right]$$

$$= \frac{3\pi^2}{4} - \frac{\pi}{2}\left[\frac{\pi}{2} - \frac{1}{2}\sin\left(2\left(\frac{\pi}{2}\right)\right) - \left(0 - \frac{1}{2}\sin(0)\right)\right] + 2\pi\left[-1\right]$$

$$= \frac{3\pi^2}{4} - \frac{\pi^2}{4} + \frac{\pi}{4}\sin(\pi) - 0 - 2\pi$$

$$= \frac{\pi^2 - \pi^2}{2} - 2\pi =$$

$$= \pi\int_0^{\frac{\pi}{2}}\left(3 - \sin^2(x) - 2\sin(x)\right)dx =$$

$$= \pi\int_0^{\frac{\pi}{2}}\left(3 - \left(\frac{1-\cos(2x)}{2}\right) - 2\sin(x)\right)dx = \pi\int_0^{\frac{\pi}{2}}\left(3 - \frac{1}{2} + \frac{1}{2}\cos(2x) - 2\sin(x)\right)dx$$

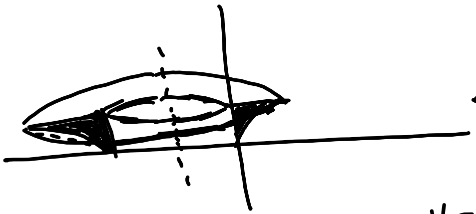
$$= \pi\int_0^{\frac{\pi}{2}}\left(\frac{5}{2} + \frac{1}{2}\cos(2x) - 2\sin(x)\right)dx = \pi\left[\frac{5}{2}x + \frac{1}{4}\sin(2x) + 2\cos(x)\right]_0^{\frac{\pi}{2}}$$

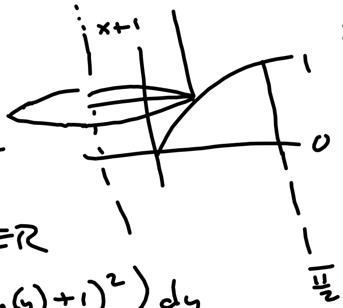
$$= \pi\left[\frac{5}{2}\left(\frac{\pi}{2}\right) + \frac{1}{4}\sin\left(2\left(\frac{\pi}{2}\right)\right) + 2\cos\left(\frac{\pi}{2}\right) - \left(\frac{5}{2}(0) + \frac{1}{4}\sin(2(0))\right) + 2\cos(0)\right]$$

$$= \pi\left[\frac{5\pi}{4} + 0 + 0 - 0 - 0 - 2(1)\right]$$

$$= \boxed{\frac{5\pi^2}{4} - 2\pi}$$

(j) Bonus 5pts Revolve the region about $x=-1$





$x = g(y) = \arcsin(y)$

$(x_R)^2-(x_L)^2$

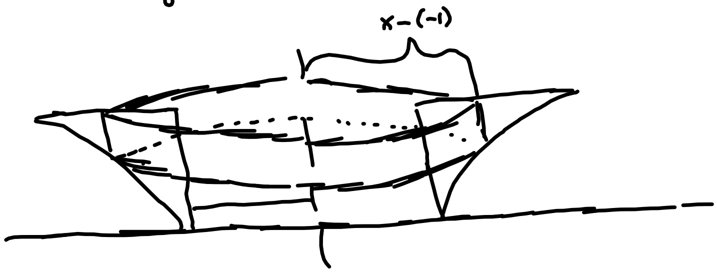
$=\left(\frac{\pi}{2}\right)^2-(\arcsin(y)+1)^2$

WASHER

$$\text{Volume} = \pi\int_0^1 \left(\left(\frac{\pi}{2}\right)^2-(\arcsin(y)+1)^2\right)dy$$

Shell:

$$2\pi\int_0^{\frac{\pi}{2}}(x+1)(1-\sin(x))\,dx$$



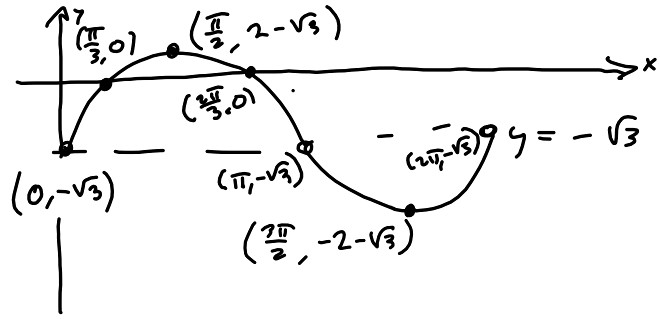
$$\left(\frac{d}{dx}[\arcsin(x)] = \frac{1}{-\cos(\arcsin(x))} = \frac{1}{-\sqrt{1-x^2}}\right)$$

3

a) 5pts

Sketch

$f(x) = 2\sin(x) - \sqrt{3}$



$2\sin(x) = \sqrt{3}$

$\sin(x) = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

b) 5pts

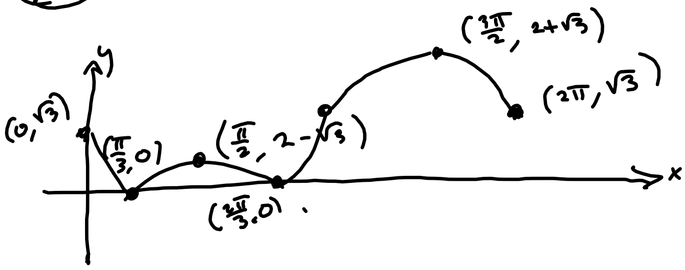
$$\int_0^{2\pi} f(x)\,dx$$

$$= \int_0^{2\pi} (2\sin(x) - \sqrt{3})\,dx = \Big[-2\cos(x) - \sqrt{3}\,x\Big]_0^{2\pi}$$

$$= -2\cos(2\pi) - \sqrt{3}(2\pi) - (-2\cos(0) - 0) = -2 - 2\pi\sqrt{3} + 2$$

$$= \boxed{-2\pi\sqrt{3}}$$

c) 5pts Sketch $|f(x)| = |2\sin(x) - \sqrt{3}|$



d) 5pts

$$-\int_0^{\frac{\pi}{3}} (2\sin(x) - \sqrt{3})\,dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (2\sin(x) - \sqrt{3})\,dx - \int_{\frac{2\pi}{3}}^{2\pi} (2\sin(x) - \sqrt{3})\,dx$$

$$= \Big[2\cos(x) + \sqrt{3}\,x\Big]_0^{\frac{\pi}{3}} + \Big[-2\cos(x) - \sqrt{3}\,x\Big]_{\frac{\pi}{3}}^{\frac{2\pi}{3}} - \Big[-2\cos(x) - \sqrt{3}\,x\Big]_{\frac{2\pi}{3}}^{2\pi}$$

$$= 2\cos\left(\tfrac{\pi}{3}\right) + \sqrt{3}\left(\tfrac{\pi}{3}\right) - (2\cos(0) + 0) + \left[-2\cos\left(\tfrac{2\pi}{3}\right) - \sqrt{3}\left(\tfrac{2\pi}{3}\right) - \left(-2\cos\left(\tfrac{\pi}{3}\right) - \sqrt{3}\left(\tfrac{\pi}{3}\right)\right)\right]$$

$$-\left[-2\cos(2\pi) - \sqrt{3}(2\pi) - \left(-2\cos\left(\tfrac{2\pi}{3}\right) - \sqrt{3}\left(\tfrac{2\pi}{3}\right)\right)\right]$$

$$= 2\left(\tfrac{1}{2}\right) + \frac{\sqrt{3}\pi}{3} - 2 + \left[-2\left(-\tfrac{1}{2}\right) - \sqrt{3}\left(\tfrac{2\pi}{3}\right) - \left(-2\left(\tfrac{1}{2}\right) - \frac{\pi\sqrt{3}}{3}\right)\right]$$

$$- \left[-2 - 2\pi\sqrt{3} + 2\left(-\tfrac{1}{2}\right) + \frac{2\pi\sqrt{3}}{3}\right]$$

$$= 1 + \frac{\sqrt{3}\pi}{3} - 2 + 1 - \frac{2\pi\sqrt{3}}{3} + 1 + \frac{\pi\sqrt{3}}{3} + 2 + 2\pi\sqrt{3} + 1 - \frac{2\pi\sqrt{3}}{3}$$

$$3 + \frac{6\pi\sqrt{3} - 2\pi\sqrt{3}}{3} + 1 = 4 + \frac{4\pi\sqrt{3}}{3}$$

4 (a) 5pts $\int \frac{1}{(2x-7)^4}\,dx = \frac{1}{2}\int (2x-7)^{-4}(2\,dx) = \frac{1}{2}\left(\frac{(2x-7)^{-3}}{-3}\right) + C$

$= \boxed{-\frac{1}{6}(2x-7)^{-3} + C}$

(b) 5pts $\int x\,(2x-7)^{-5}\,dx$

$u = 2x-7 \Longrightarrow x = \frac{u+7}{2}$

$du = 2\,dx$

$\frac{du}{2} = dx$

$= \int \left(\frac{u+7}{2}\right)(u^{-5})\left(\frac{du}{2}\right) = \frac{1}{4}\int (u+7)u^{-5}\,du = \frac{1}{4}\int (u^{-4}+7u^{-5})\,du$

$= \frac{1}{4}\left(\frac{u^{-3}}{-3} - \frac{7u^{-4}}{-4}\right) + C = \boxed{-\frac{1}{12}(2x-7)^{-3} + \frac{7}{16}(2x-7)^{4} + C}$

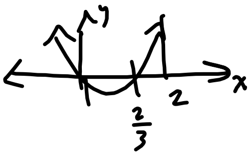
(c) 5pts $\int \sin^5(x)\cos^3(x)\,dx = \int \sin^5(x)(1-\sin^2(x))\cos(x)\,dy$

$= \int \sin^5(x)\cos(x)\,dx - \int \sin^7(x)\cos(x)\,dx = \boxed{\frac{\sin^6(x)}{6} - \frac{\sin^8(x)}{8} + C}$

(d) 5pts $\int 2^{3x}\,dx = \int e^{3\ln(2)x}\,dx = \frac{1}{3\ln(2)}e^{3\ln(2)x} + C$

$= \boxed{\frac{1}{3\ln(2)}\cdot 2^{3x} + C}$

5 $f'(x) = 3x^2 - 2x = x(3x-2)$



a Net change over

Net Change of f is $\int f'(x)\,dx = \left[x^3 - x^2\right]_0^2 = 8 - 4 = 4$ = net change

b Total change in f over $[0,2]$

Total Change: $\int_0^2 |f'(x)|\,dx = \int_0^2 |3x^2-2x|\,dx = -\int_0^{\frac{2}{3}} (3x^2-2x)\,dx$

$+ \int_{\frac{2}{3}}^2 (3x^2-2x)\,dx = -\left[x^3 - x^2\right]_0^{\frac{2}{3}} + \left[x^3 - x^2\right]_{\frac{2}{3}}^2$

$= -\left[\left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2 - (0-0)\right] + \left[2^3 - 2^2 - \left[\left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2\right]\right]$

$= \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 + 8 - 4$

$= 2\left[\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3\right] + 4 = 2\left[\frac{12-8}{27}\right] + 4$

$= \frac{8}{27} + \frac{108}{27} = \boxed{\frac{116}{27} = \text{Total Change}}$

6 a 5pts $y = 3\cdot 7^{\sqrt{x}} \longrightarrow y' = \ln(7)\left(\frac{1}{2}\right)x^{-\frac{1}{2}}\cdot 3\cdot 7^{\sqrt{x}} = \boxed{\frac{3\ln(7)}{2\sqrt{x}}\cdot 7^{\sqrt{x}}}$

b 5pts $y = \ln\left(\frac{\tan^7(x)\sqrt{x^2+1}}{\sqrt[13]{\cos(5x)}}\right)$

$= 7\ln(\tan(x)) + \frac{1}{2}\ln(x^2+1) - \frac{1}{13}\ln(\cos(5x)) \longrightarrow y' =$

$= \boxed{7\frac{\sec^2(x)}{\tan(x)} + \frac{1}{2}\left(\frac{2x}{x^2+1}\right) - \frac{1}{13}\left(\frac{-5\sin(5x)}{\cos(5x)}\right) = y'}$

c 5pts $y = \log_7(x\sin(5x)) = \log_7(x) + \log_7(\sin(5x))$

$\longrightarrow \boxed{y' = \frac{1}{\ln(7)x} + \frac{1}{\ln(7)}\left(\frac{5\cos(5x)}{\sin(5x)}\right)}$

d 5pts $y = [x^2+5x]^{\tan(x)} \longrightarrow$

$\ln(y) = \tan(x)\ln(x^2+5x) \longrightarrow$

$\frac{y'}{y} = \sec^2(x)\ln(x^2+5x) + (\tan(x))\left(\frac{2x+5}{x^2+5x}\right) \longrightarrow$

$\boxed{y' = \left[\sec^2(x)\ln(x^2+5x) + (\tan(x))\left(\frac{2x+5}{x^2+5x}\right)\right][x^2+5x]^{\tan(x)}}$

e 5pts $y = \int_0^x \frac{\sin(5t)+t^3}{\sqrt{\cos^2(t^2)+1}}\,dt \longrightarrow$

$\boxed{y' = \frac{\sin(5x)+x^3}{\sqrt{\cos^2(x^2)+1}}}$

f $y = \int_0^{2x^2-3x} \frac{\sin(5t)+t^3}{\sqrt{\cos^2(t^2)+1}}\,dt \longrightarrow$

$\boxed{y' = \left(\frac{\sin(5(2x^2-3x)) + (2x^2-3x)^3}{\sqrt{\cos^2((2x^2-3x)^2)+1}}\right)(4x-3)}$

(7) $\cancel{f(x)}= x^2+6x-27 = g^{-1}(x)$

$x = y^2+6y-27 = x$

$y^2+6y+3^2 = x+27+9$

$(y+3)^2 = x+36$

$y+3 = \pm\sqrt{x+36}$

$y = -3\pm\sqrt{x+36}$

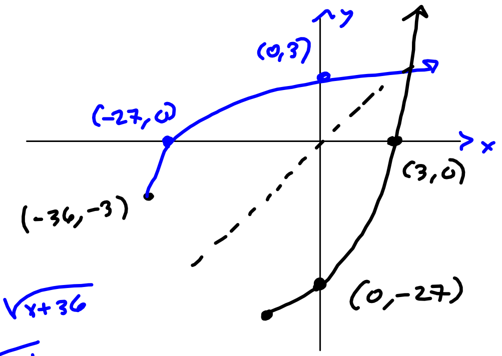
(7) Let $f(x) = \sqrt{x+36} - 3$. $\overset{SET}{=0} \Rightarrow x+36 = 9 \Rightarrow x = -27$

(a) (5pts) What is its domain and range?

$\mathcal{D}(f) = [-36,\infty)$

$\mathcal{R}(f) = [-3,\infty)$

$f(0) = \sqrt{36} - 3$
$= 6-3 = 3$



$y = -3+\sqrt{x+36}$

$y+3 = \sqrt{x+36}$

$(y+3)^2 = x+36$

$x = (y+3)^2-36$

$f^{-1}(x) = (x+3)^2-36$

$(f^{-1})' = 2(x+3)$

$(f^{-1})'(2) = 2(5) = 10$

By Theorem:

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))}$$

$f(x) = 2$

$\sqrt{x+36} - 3 = 2 \to x+36 = 5^2 \Rightarrow x = -36+25 = -11 = f^{-1}(2)$

$$f'(x) = \tfrac{1}{2}(x+36)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x+36}}$$

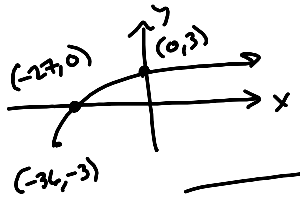
$$f'(f^{-1}(2)) = f'(-11) = \frac{1}{2\sqrt{-11+36}} = \frac{1}{2\sqrt{25}} = \frac{1}{2\cdot 5} = \frac{1}{10}$$

$$\Rightarrow (f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{\frac{1}{10}} = 10 \checkmark$$

7 $f(x)=\sqrt{x+36}-3 \overset{\text{SET}}{=} 0 \Rightarrow \sqrt{x+36}=3$

$\Rightarrow x+36=9$

$\Rightarrow x=9-36=-27$



a 5pts $\boxed{\mathcal{D}(f)=[-36,\infty) \text{ \& } \mathcal{R}(f)=[-3,\infty)}$

b 5pts $\sqrt{y+36}-3=x$

$\sqrt{y+36}=x+3$

$y+36=(x+3)^2$

$y=\boxed{(x+3)^2-36=f^{-1}(x)}$

$\boxed{\mathcal{D}(f^{-1})=[-3,\infty) \text{ \& } \mathcal{R}(f^{-1})=[-36,\infty)}$

c 5pts $\boxed{(f^{-1})'(x)=2(x+3)}$

d 5pts $(f^{-1})'(2)=2(2+3)=\boxed{10=(f^{-1})'(2)}$

e 5pts $(f^{-1})'(2)=\dfrac{1}{f'(f^{-1}(2))}=\dfrac{1}{\quad}=\dfrac{1}{\quad}$

Scratch: $f^{-1}(2)$: $f(x)\overset{\text{SET}}{=}2 \Rightarrow$

$\sqrt{x+36}-3=2 \Rightarrow$

$\sqrt{x+36}=5 \Rightarrow$

$x+36=25 \Rightarrow$

$x=25-36=-11=f^{-1}(2)$

$f^{-1}(2)=(2+3)^2-36=25-36=-11$ ✓

$f'(x)=\frac{1}{2}(x+36)^{-\frac{1}{2}}$

$\dfrac{1}{f'(-11)}=\dfrac{1}{\frac{1}{2}(-11+36)^{-\frac{1}{2}}}=\dfrac{1}{\frac{1}{2}(25)^{-\frac{1}{2}}}\ \dfrac{1}{\frac{1}{10}}=\boxed{10=(f^{-1})'(2)}$

(8) (5pts) Let $h = r$ = height or radius of the conical pile of sand that is being added to at a rate of $30 \frac{ft^3}{min}$. Units in feet. We want

$$\left.\frac{dh}{dt}\right|_{h=10}$$

V = volume in $ft^3 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3 \implies$

$$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt} \implies \frac{dh}{dt} = \frac{1}{\pi h^2}\frac{dV}{dt} \implies$$

$$\left.\frac{dh}{dt}\right|_{h=10} = \frac{1}{\pi(10)^2} \cdot 30 = \frac{30}{100\pi} = \boxed{\frac{3}{10\pi}\ ft/min}$$

$$\approx 0.0954929 6583\ ft/min$$

9 5pts Claim: $\lim_{x \to 5} (2x-3) = 7$

Proof: Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{2}$.

If $0 < |x-5| < \delta$, then $|2x-3-7| = |2x-10| = 2|x-5|$

$< 2\delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon$ ■

10 $x^2+y^2-2xy = \sin(\pi xy) \Longrightarrow$

$2x+2yy'-2y-2xy' = \pi\cos(\pi xy)(y+xy') =$

$\Longrightarrow \qquad = \pi y \cos(\pi xy) + (\pi x \cos(\pi xy))y'$

$\Longrightarrow (2y-2x-\pi x\cos(\pi xy))y' = 2y-2x+\pi y\cos(\pi xy)$

$\Longrightarrow \boxed{y' = \dfrac{2y-2x+\pi y\cos(\pi xy)}{2y-2x-\pi x\cos(\pi xy)}}$

B1 5pts $y'\Big|_{(1,1)} = \dfrac{2(1)-2(1)+\pi(1)\cos(\pi)}{2(1)-2(1)-\pi(1)(\cos(\pi)} = \dfrac{-\pi}{-\pi(-1)} = -1$

$\Longrightarrow \boxed{y = -1(x-1)+1}$ is tangent to the graph of #10.

B2 5pts **Claim:** $\lim_{x \to 5} (x^2-3) = 22$

Scratch: want $|x^2-3-22| = |x-5||x+5| < \varepsilon$

$|x-5| < \delta$

Need bound on $|x+5|$.

Assume $\delta \leq 1$. Then $x \to 5$

$\Rightarrow 4 < x < 6$

$\Rightarrow 4+5 < x+5 < 6+5$

$9 < x+5 < 11$

$\Rightarrow |x+5| < 11$

Proof Let $\varepsilon > 0$ be given. Define $\delta = \min\left\{1, \frac{\varepsilon}{11}\right\}$.

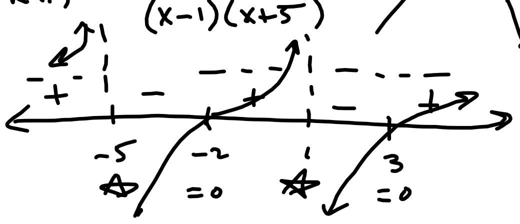
Then $0 < |x-5| < \delta$ implies

$|x^2-3-22| = |x+5||x-5| < 11\delta \leq 11 \cdot \frac{\varepsilon}{11} = \varepsilon$ ■

11 $R(x) = \dfrac{(x+2)(x-3)}{(x-1)(x+5)} = y$

$\boxed{y = 1 \text{ H.A.}}$

$\boxed{x = 1,\ x = -5 \text{ V.A}}$



B3 5pts

$$y = \frac{(x+2)(x-3)}{(x-1)(x+5)}$$

$$\ln y = \ln(x+2) + \ln(x-3) - \ln(x-1) - \ln(x+5)$$

$$\Longrightarrow \frac{y'}{y} = \frac{1}{x+2} + \frac{1}{x-3} - \frac{1}{x-1} - \frac{1}{x+5}$$

$$\Longrightarrow \boxed{y' = \left(\frac{1}{x+2} + \frac{1}{x-3} - \frac{1}{x-1} - \frac{1}{x+5}\right)\left(\frac{(x+2)(x-3)}{(x-1)(x+5)}\right)}$$ Stop!

$$= \frac{(x-3)}{(x-1)(x+5)} + \frac{x+2}{(x-1)(x+5)} - \frac{(x+2)(x-3)}{(x-1)^2(x+5)} - \frac{(x+2)(x-3)}{(x-1)(x+5)^2}$$

Bonus 4

5pts

$R(x) = \dfrac{(x+3)(x-3)}{x-5}$

$\boxed{\text{V.A.: } x=5}$

$\boxed{\begin{array}{l}\text{x-int: } (-3,0), (3,0) \\ \text{y-int: } (0, -\frac{9}{5})\end{array}}$

SLANT:

$\dfrac{x^2-9}{x-5}$:

$$\begin{array}{r|rrr} 5 & 1 & 0 & -9 \\ & & 5 & \\ \hline & 1 & 5 & \end{array}$$

$\boxed{\text{S.A.: } y = x+5}$

$$R'(x) = \frac{2x(x-5) - (x^2-9)(1)}{(x-5)^2} = \frac{2x^2 - 10x - x^2 + 9}{(x-5)^2} = \boxed{\begin{array}{c} R'(x) \\ \dfrac{x^2-10x+9}{(x-5)^2}\end{array}} \text{ SET } 0 \Rightarrow$$

$(x-9)(x-1) = 0 \rightarrow x \in \{1, 9\}$

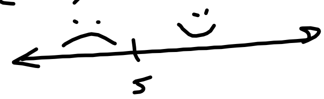
$R(1) = \dfrac{1^2-9}{1-5} = \dfrac{-8}{-4} = 2 \rightarrow \boxed{(1,2) \;\; R'=0}$

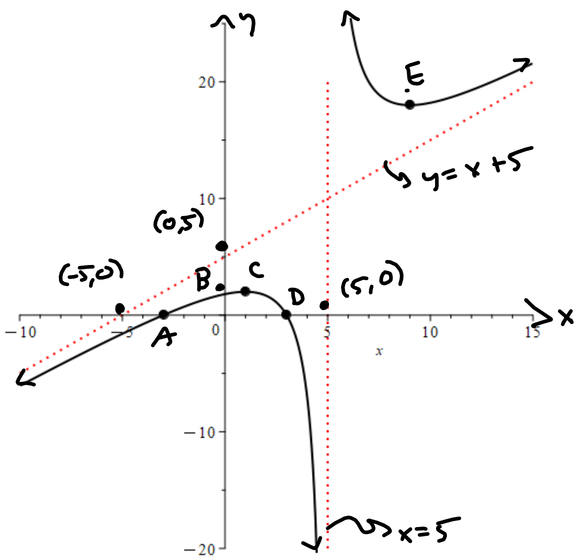
$R(9) = \dfrac{9^2-9}{9-5} = \dfrac{81-9}{4} = \dfrac{72}{4} = \dfrac{36}{2} = 18 \rightarrow \boxed{(9,18) \; R'=0}$

$$R''(x) = \frac{(2x-10)(x-5)^2 - (x^2-10x+9)(2(x-5))}{(x-5)^4}$$

$$= \frac{(x-5)\left((2x-10)(x-5) - 2(x^2-10x+9)\right)}{(x-5)^4} = \frac{2x^2 - 10x - 10x + 50 - 2x^2 + 20x - 18}{(x-5)^3}$$

$$= \frac{32}{(x-5)^3}$$





A = (-3,0) x-int
B = $(0, \frac{9}{5})$ y-int
C = (1,2) MAX
D = (3,0) x-int
E = (9,18) MIN