Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$x^2 - xy - y^2 = 1, \quad (2, 1)$$

$y = m(x - x_1) + y_1$

$y = m(x-2) + 1$

$2x - y - xy' - 2yy' = 0$

$\rightarrow (-x-2y)y' = -2x+y$

$\rightarrow y' = \dfrac{-2x+y}{-x-2y} = \boxed{\dfrac{2x-y}{x+2y} = y'}$

$y'\Big|_{\substack{x=2\\y=1}} = \dfrac{2(2)-1}{2+2(1)} = \dfrac{3}{4} = m$

$\boxed{y = \frac{3}{4}(x-2)+1}$

Differentiate:

$$y = \frac{(x+2)^5 (2x-5)^{\frac{2}{3}}}{(x-6)\sqrt[4]{x^2-5x}}$$

$$\ln(y) = 5\ln(x+2) + \frac{2}{3}\ln(2x-5) - \ln(x-6) - \frac{1}{4}\ln(x^2-5x)$$

$$\Rightarrow \frac{y'}{y} = 5\left(\frac{1}{x+2}\right) + \frac{2}{3}\left(\frac{2}{2x-5}\right) - \frac{1}{x-6} - \frac{1}{4}\left(\frac{2x-5}{x^2-5x}\right) \longrightarrow$$

$$\boxed{y' = \left(\frac{5}{x+2} + \frac{4}{3(2x-5)} - \frac{1}{x-6} - \frac{2x-5}{x^2-5x}\right)\left(\frac{(x+2)^5(2x-5)^{\frac{2}{3}}}{(x-6)\sqrt[4]{x^2-5x}}\right)}$$

Sketch a complete graph

$$y = \frac{(x+2)(x-3)}{x+1} = \frac{x^2-x-6}{x+1}$$

$D = \mathbb{R} \setminus \{-1\}$

$\boxed{\text{V.A.: } x=-1}$

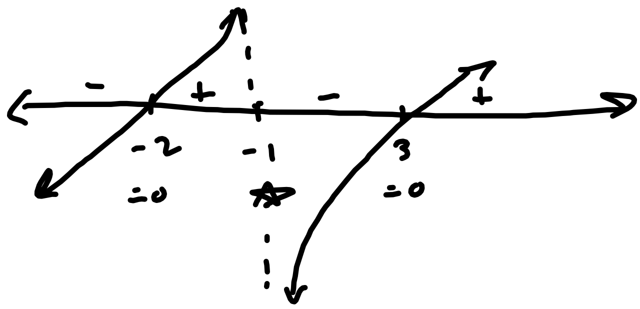
$\boxed{\text{x-int: } (-2,0), (3,0)}$

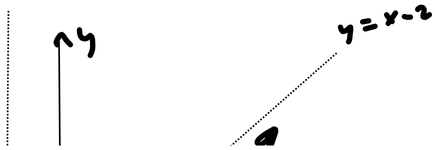
y-int: $\frac{(2)(-3)}{1} = -6 \rightsquigarrow \boxed{(0,-6)}$

O.A.:

$$\begin{array}{r|rrr} -1 & 1 & -1 & -6 \\ & & -1 & 2 \\ \hline & 1 & -2 & \end{array}$$

$\boxed{\text{O.A.: } y = x-2}$





Approximate $\sqrt{99}$ with a differential.

$x_1 = 100$

$x_2 = x_1 + \Delta x = x_1 + dx = 100 - 1 \longrightarrow dx = -1 = \Delta x$

$f(x) = \sqrt{x}$ want $f(99) = f(x_1 + \Delta x) \approx f(x_1) + f'(x_1)dx$

$f(100) = \sqrt{100} = 10$ $= 10 + f'(100)(-1)$

$f(x) = x^{\frac{1}{2}} \longrightarrow f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ $f(100-1) = f(x_1 + \Delta x)$

$\longrightarrow f'(x_1) = \frac{1}{2}(100)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{1}{10}\right) = \frac{1}{20} \longrightarrow$

$\sqrt{99} \approx 10 + \left(\frac{1}{20}\right)(-1) = 10 - \frac{1}{20}$

$= \frac{200-1}{20} = \boxed{\frac{199}{20}}$ A smidge less than 10, as expected.

Equivalent Method:

Use tangent-line approximation.