One antiderivative we don't know:

$\int \ln(x)\,dx$. But we DO know $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

$(uv)' = u'v + uv'$

$-uv' = -(uv)' + u'v =$

$uv' = (uv)' - vu'$

$\int uv' = \int (uv)' - \int vu'$

$\int u(x)v'(x)\,dx = \int d(uv) - \int v(x)u'(x)\,dx$

$\underline{\int u\,dv = uv - \int v\,du}$ Integration by Parts

$\int \ln(x)\,dx = ?$

$u = \ln(x) \Rightarrow du = \frac{1}{x}\,dx$

$dv = dx \Rightarrow v = x$

$\Rightarrow uv - \int v\,du = \ln(x)\cdot x - \int x\cdot\frac{1}{x}\,dx$ $\int \ln(x)\,dx =$

$= x\ln(x) - \int dx = \boxed{x\ln(x) - x + C}$

$\int x^n \sin(x)\,dx$

$u = x^n$

$dv = \sin(x)\,dx$

$f(x) = \sqrt{x+36} - 3$

Prove f is 1-to-1:

$(x_1, f(x_1)) = (x_2, f(x_2))$

if $f(x_1) = f(x_2)$

S $f(x_1) = f(x_2)$

$\sqrt{x_1+36} - 3 = \sqrt{x_2+36-3}$

$\sqrt{x_1+36} = \sqrt{x_2+36}$

$x_1 + 36 = x_2 + 36$

$x_1 = x_2$

Find the inverse function

$\sqrt{y+36} - 3 = x$

$\sqrt{y+36} = x+3$

$y+26 = (x+3)^2$

$y = (x+3)^2 - 36 = f^{-1}(x)$

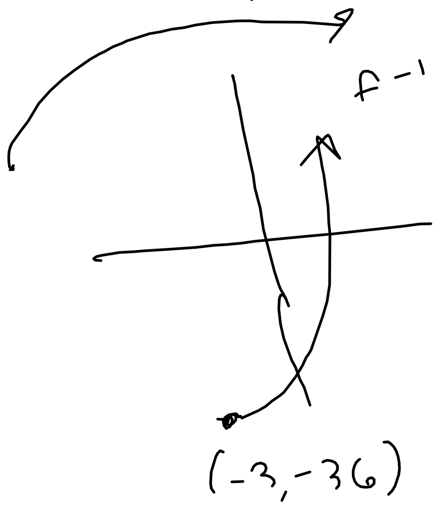
$D(f) = [-36, \infty)$ from needing $x+36 \geq 0$

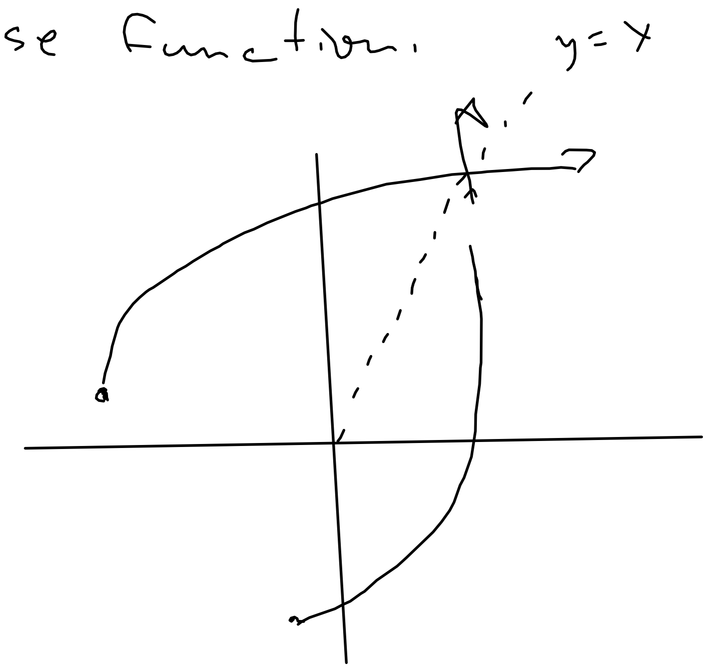
$R(f) = [-3, \infty)$

$D(f) = R(f^{-1})$

$R(f) = D(f^{-1})$

But $D((x+3)^2 - 36) = \mathbb{R}$, but we restrict its domain so that $f(x)$ is its inverse function.





$(f^{-1})'(2)$ directly

$$(f^{-1})' = \frac{d}{dx}\left[(x+3)^2 - 36\right] = 2(x+3)$$

$$\Rightarrow (f^{-1})'(2) = 2(2+3) = 10 = (f^{-1})'(2)$$

FORMULA: $$(f^{-1})'(2) = \frac{1}{f'(\underbrace{f^{-1}(2)}_{\text{Need}})}$$

$$f'(x) = \frac{d}{dx}\left[(x+36)^{\frac{1}{2}} - 3\right]$$

$$= \frac{1}{2}(x+36)^{-\frac{1}{2}}(1)$$

Find $f^{-1}(2)$ by solving $f(x) = 2$

$$\sqrt{x+36} - 3 \overset{\text{SET}}{=} 2$$

$$\sqrt{x+36} = 5$$

$$x + 36 = 25$$

$$\boxed{x = -11} = f^{-1}(2)$$

$$\frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(-11)}$$

$$= \frac{1}{\frac{1}{10}} \; \boxed{\begin{aligned} &= 10 \checkmark \\ &= (f^{-1})'(2) \end{aligned}}$$

$$f'(-11) = \frac{1}{2}(-11+36)^{-\frac{1}{2}}$$

$$= \frac{1}{2}(25)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1}{25^{\frac{1}{2}}}\right) = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$$

1-to-1 means

if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$

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if $f(x_1) = f(x_2)$ then $x_1 = x_2$

If A then B is the same as If "not B" then "not A"

#1 $f(x) = x^2 - 5x$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5(x+h) - (x^2 - 5x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h} = \frac{2xh + h^2 - 5h}{h}$$

$$= \frac{h(2x + h - 5)}{h} \xrightarrow{h \to 0} \boxed{2x - 5 = f'(x)}$$

(b) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} (x^2 - 5x)\,dx$ by the definition.

$$= \int_{-1}^{1} x^2\,dx - \int_{-1}^{1} 5x\,dx = \int_{-1}^{1} x^2\,dx$$! (5x is odd and [-1,1] is a symmetric interval)

$$\Delta x = \frac{b-a}{n} = \frac{1-(-1)}{n} = \frac{2}{n}$$

$$x_k = a + \frac{b-a}{n} k = -1 + \frac{2}{n} k = -1 + \frac{2k}{n}$$

$$= \frac{2k}{n} - 1 = \frac{2k - n}{n}$$

$$A \xrightarrow{h \to 0} B$$

$$\frac{b-a}{n} \sum_{k=1}^{n} f(x_k) = \frac{2}{n} \sum_{k=1}^{n} x_k^2 = \frac{2}{n} \sum_{k=1}^{n} \left(\frac{2k}{n} - 1\right)^2$$

$$= \frac{2}{n} \sum_{k=1}^{n} \left(\left(\frac{2k}{n}\right)^2 - 2\left(\frac{2k}{n}\right)(1) + 1^2 \right)$$

$$= \frac{2}{n} \sum_{k=1}^{n} \left(\frac{4k^2}{n^2} - \frac{4k}{n} + 1 \right) = \frac{2}{n} \left[\sum_{k=1}^{n} \frac{4k^2}{n^2} - \sum_{k=1}^{n} \frac{4k}{n} + \sum_{k=1}^{n} 1 \right]$$

$$= \frac{2}{n} \cdot \frac{4}{n^2} \sum_{k=1}^{n} k^2 - \frac{2}{n} \cdot \frac{4}{n} \sum_{k=1}^{n} k + \frac{2}{n} \cdot n$$

$$= \frac{8}{n^3} \cdot \frac{n^3 + \cdots}{3} - \frac{8}{n^2} \cdot \frac{n^2 + \cdots}{2} + 2$$

$$\xrightarrow{n \to \infty} \frac{8n^3}{3n^3} - \frac{8n^2}{2n^2} + 2$$

$$= \frac{8}{3} - 4 + 2 = \frac{8}{3} - 2 = \frac{8-6}{3} = \boxed{\frac{2}{3} = \int_{-1}^{1} f(x)\,dx}$$

Check with FTC II

$\int_{-1}^{1} (x^2 - 5x)\,dx = \int_{-1}^{1} x^2\,dx$, b/c $5x$ is odd & [-1,1] is symmetrix

$$= \left.\frac{x^3}{3}\right]_{-1}^{1} = \frac{1^3}{3} - \left(\frac{(-1)^3}{3}\right) = \frac{1}{3} - \left(-\frac{1}{3}\right) = \boxed{\frac{2}{3} = \int_{-1}^{1} f(x)\,dx}$$