5. Let f be a smooth function such that $f'(x) = 3x^2 - 2x$

a. (5 pts) What's the net change in f over the interval $[0,2]$? ?

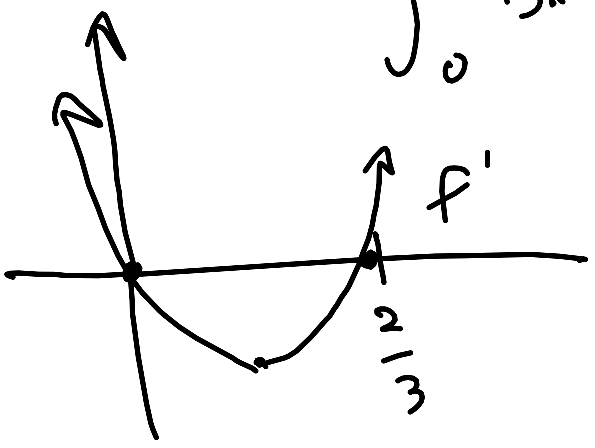
$$\int_0^2 (3x^2 - 2x)\,dx = x^3 - x^2\Big]_0^2 = f(2) - f(0) =$$

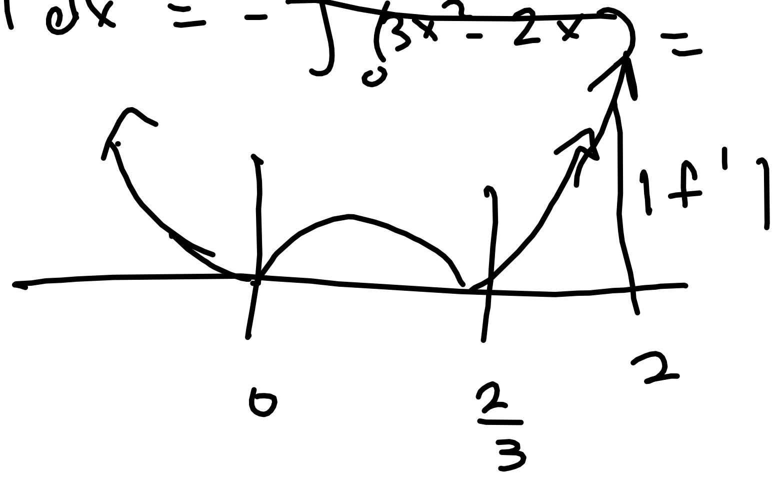
$\boxed{=4}$ = net.

$$3x - 2 = 0 \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$$

(5 pts) What's the *total* change in f over the interval $[0,2]$?

$$\int_0^2 |3x^2 - 2x|\,dx = -\cancel{\int_0^2 (3x^2 - 2x)} =$$





Total change is

$$-\int_0^{2/3} (3x^2 - 2x)\,dx + \int_{2/3}^2 (3x^2 - 2x)\,dx$$

$$= -(x^3 - x^2)\Big|_0^{2/3} + \left[x^3 - x^2\right]_{2/3}^2 = -\left(\frac{8}{27} - \frac{4}{9}\right) + \left(8 - 4 - \left(\frac{8}{27} - \frac{4}{9}\right)\right)$$

$$= -\left(\frac{8-12}{27}\right) + 4 - \left(\frac{8-12}{27}\right)$$

$$= \frac{4}{27} + 4 - \left(-\frac{4}{27}\right)$$

$$= \frac{8}{27} + 4\left(\frac{27}{27}\right) = \frac{8+108}{27}$$

$$= \boxed{\frac{116}{27}} \approx \boxed{4.2962962963}$$

= Total change.

4. Evaluate the indefinite integrals. $\boxed{\textbf{Do Not Simplify}}$.

a. (5 pts) $\int \frac{1}{(2x-7)^4}\,dx$

$u = 2x-7$

$du = 2\,dx$

$\boxed{dx = \frac{du}{2}}$

$= \int u^{-4}\,\frac{du}{2} = \boxed{\frac{1}{2}\,\frac{u^{-3}}{-3} + C}$

$\frac{\frac{1}{2}(2x-7)^{-3}}{-3} + C$ STOP!

$= -\frac{1}{6}(2x-7)^{-3} + C$

b. (5 pts) $\int x(2x-7)^{-5}\,dx$

$u = 2x-7$

$du = 2\,dx$

$\frac{du}{2} = dx$

$= \int x\,(u)^{-5}\,\frac{du}{2}$

$2x-7 = u$

$2x = u+7$

$x = \frac{u+7}{2}$

$= \int \left(\frac{u+7}{2}\right)(u)^{-5}\,\frac{du}{2}$

$= \frac{1}{2}\int \left(u^{-4} + 7u^{-5}\right)du$

$= \boxed{\frac{1}{2}\,\frac{(2x-7)^{-3}}{-3} + 7\,\frac{(2x-3)^{-4}}{-4} + C}$

c. (5 pts) $\int \sin^5(x)\cos^3(x)\,dx$

$= \int \sin^5(x)\cos^2(x)\cos(x)\,dx$

$= \int \sin^5(x)\left(1-\sin^2(x)\right)\cos(x)\,dx$

$= \int \sin^5(x)\cos(x) - \sin^7(x)\cos(x)\,dx$

$u = \sin(x)$

$du = \cos(x)\,dx$

$= \int u^5\,du - \int u^7\,du$

$= \frac{u^6}{6} - \frac{u^8}{8} + C = \frac{1}{6}\sin^6(x) - \frac{1}{8}\sin^8(x) + C$

d. (5 pts) $\int 2^{3x}\,dx$

$u = 3x$

$du = 3\,dx$

$dx = \frac{du}{3}$

$= \frac{1}{3}\int 2^{3x}\,3\,dx$

$= \int 2^u\,\frac{du}{3}$

$= \frac{1}{\ln(2)}\,2^u + C \to \frac{1}{\ln(2)}\cdot 2^{3x} + C$

$\int 2^{3x}\,dx = \int \left(e^{\ln(2)}\right)^{3x} dx = \int e^{(3\ln(2))x}\,dx$

$u = 3\ln(2)\,x$

$du = 3\ln(2)\,dx$

$\frac{du}{3\ln(2)} = dx$

$= \int e^u\,\frac{du}{3\ln(2)}$

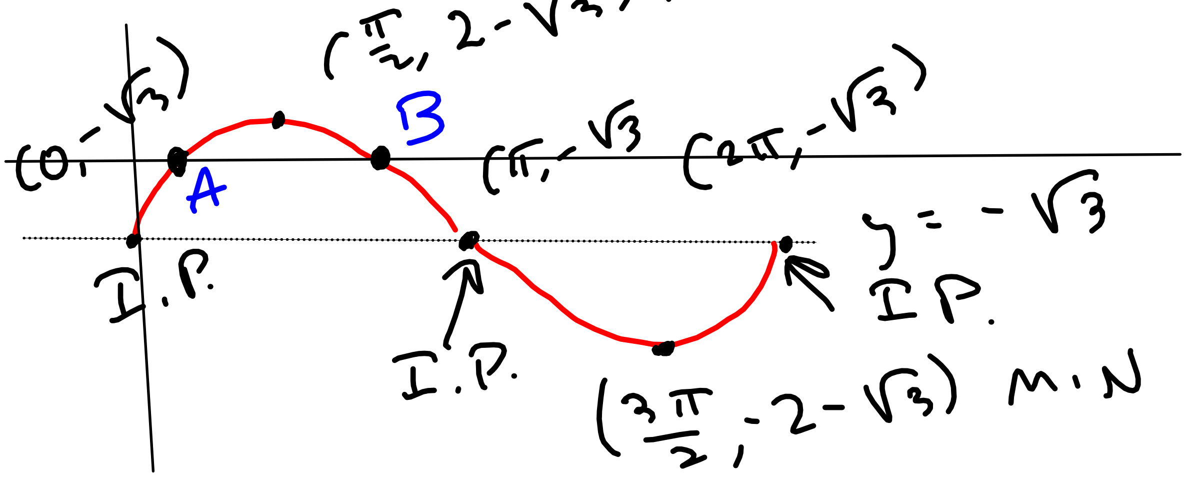
$= \frac{1}{3\ln(2)}\,e^u + C$

$= \boxed{\frac{1}{3\ln(2)}\,e^{3x} + C}$

3. We explore absolute value. Let $f(x) = 2\sin(x) - \sqrt{3}$

$y = -\sqrt{3}$ is midline
Amplitude $= 2$

a. (5 pts) Sketch a complete graph of $f(x)$ on the interval $[0, 2\pi]$. That means all intercepts, local extreme points and inflection points. This requires no calculus, if you're familiar with the graph of sine.



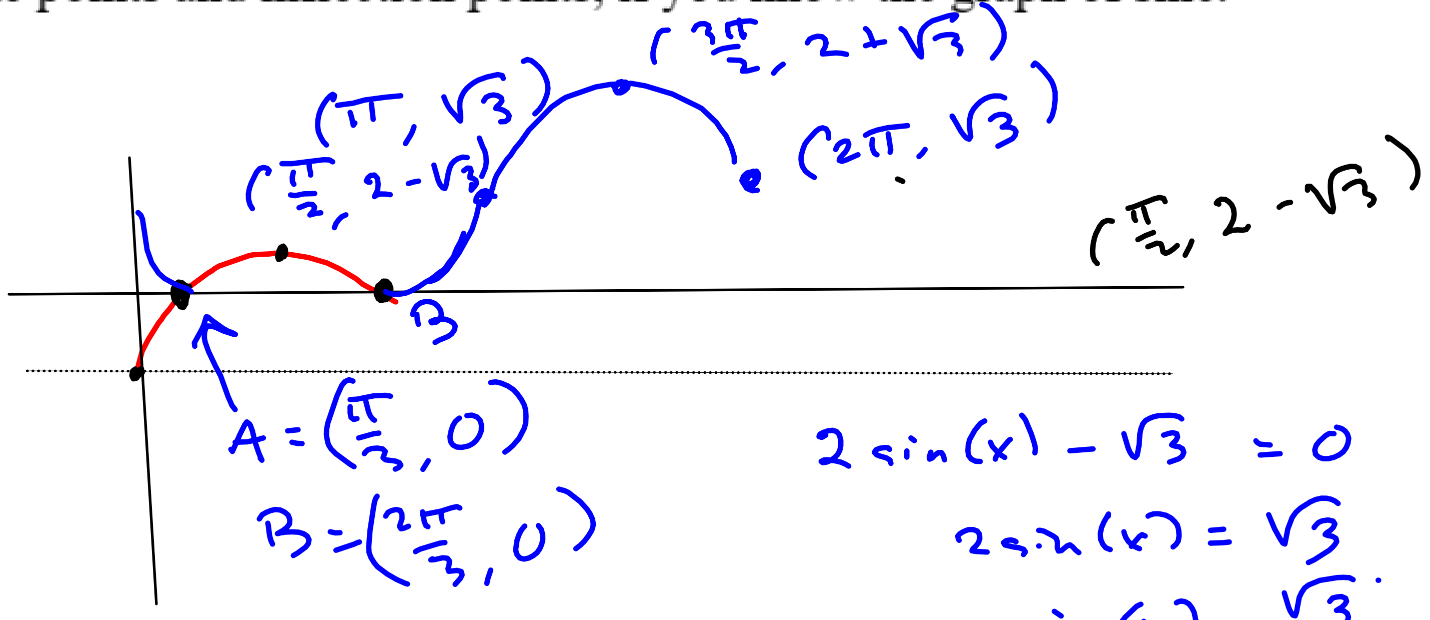
b. (5 pts) Evaluate $\int_0^{2\pi} f(x)\,dx$. $= \int_0^{2\pi} (2\sin(x) - \sqrt{3})\,dx$

$= -2\cos(x) - \sqrt{3}x \Big]_0^{2\pi}$

$= -2\cos(2\pi) - \sqrt{3}(2\pi) - (-2\cos(0) - \sqrt{3}(0))$

$= -2(1) - 2\sqrt{3}\pi + 2 = \boxed{-2\sqrt{3}\pi}$

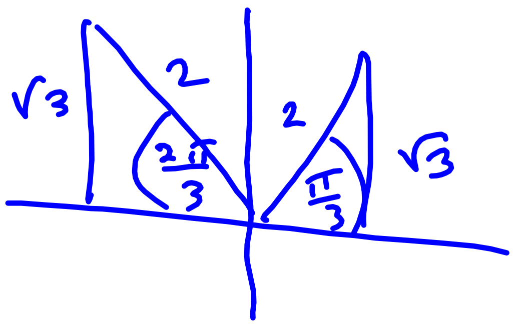
c. (5 pts) Sketch a complete graph of $|f(x)|$ on the interval $[0, 2\pi]$. It doesn't require calculus to find local extreme points and inflection points, if you know the graph of sine.



$A = (\frac{\pi}{3}, 0)$
$B = (\frac{2\pi}{3}, 0)$

$2\sin(x) - \sqrt{3} = 0$
$2\sin(x) = \sqrt{3}$
$\sin(x) = \frac{\sqrt{3}}{2}$

d. (5 pts) Evaluate $\int_0^{2\pi} |f(x)|\,dx$. $= -\int_0$



$= -\int_0^{\frac{\pi}{3}} f(x)\,dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} f(x)\,dx$

$- \int_{\frac{2\pi}{3}}^{2\pi} f(x)\,dx$

Simplifying further might not be the best use of your time on a test.

etc.

2. Consider the region bounded by $y = \sin(x)$, $y = 1$, $x = 0$, and $x = \frac{\pi}{2}$. in two ways.

a. (5 pts) Sketch the region.

(5 pts) Write the area of the region as an integral with respect to x. Draw a representative rectangle on the sketch from part a.

c. (5 pts) Evaluate the integral from part b.

d. (5 pts) Sketch the region, again.

e. (5 pts) Write the area as an integral with respect to y. Draw a representative rectangle on the sketch from #2*d*. Do not evaluate the integral.

f. (**Bonus** 5 pts) Evaluate your answer to #2*e*. Points:Time = small.

g. (5 pts) Suppose we rotated the region about the line $y = -1$. Sketch the graph of the solid o
revolution obtained. Include a representative disk or washer.

h. (5 pts) Write the integral representing the volume of the solid of revolution in #2*g*.

h. (5 pts) Write the integral representing the volume of the solid of revolution in #2*g*.

i. (**Bonus** 5 pts) Evaluate the integral you wrote for #2*h*.

j. (**Bonus** 5 pts) Write the integral for revolving the region about the line $x = -1$. Not fun.

k. (**Bonus** 5 pts) Evaluate the integral in #2*j*. (Beyond the scope of the course)