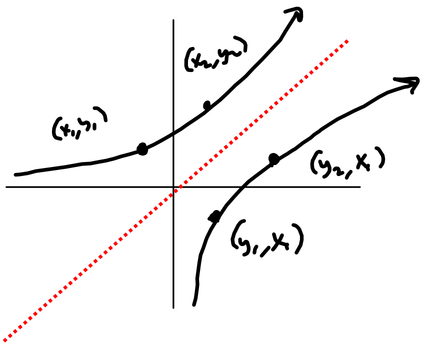
S 6.1

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$



m_{sec} for f: $\dfrac{y_2-y_1}{x_2-x_1}$

m_{sec} for f^{-1}: $\dfrac{x_2-x_1}{y_2-y_1}$? It's $\dfrac{1}{m_{sec} \text{ for } f}$

39–42 Find $(f^{-1})'(a)$.

42. $f(x) = \sqrt{x^3 + 4x + 4}, \quad a = 3$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))}$$

$f^{-1}(3)$:

$f(x) = 3$

$\sqrt{x^3+4x+4} = 3$ Usually, these we can solve by inspection or guess-and-check

$\sqrt{1^3+4(1)+4} = \sqrt{9} = 3 \implies f^{-1}(3) = 1$

So we need $f'(1)$.

$f(x) = (x^3+4x+4)^{\frac{1}{2}} \implies$

$f'(x) = \frac{1}{2}(x^3+4x+4)^{-\frac{1}{2}}(3x^2+4) \implies$

$f'(1) = \frac{1}{2}(9)^{-\frac{1}{2}}(3+4) = \frac{1}{2}\left(\frac{1}{3}\right)(7) = \frac{7}{6} \implies$

$\frac{1}{\sqrt{9}} = \frac{1}{3}$

$$\boxed{(f^{-1})'(3) = \frac{6}{7}}$$

§1.?

Find Domain:

$$f(x) = \frac{1-e^{x^2}}{1-e^{1-x^2}}$$

$\sqrt{\text{Negative}}$ is Bad.

For $\sqrt{\text{radicand}}$: Need radicand ≥ 0

$\frac{\text{STUFF}}{0}$ is bad.

For $\frac{\text{STUFF}}{\text{JUNK}}$: Need Junk $\neq 0$.

Need $1 - e^{1-x^2} \neq 0$

$$-e^{1-x^2} \neq -1$$

$$e^{1-x^2} \neq 1$$

$$\ln(e^{1-x^2}) \neq \ln(1) \qquad 1 = e^0$$

$$1 - x^2 \neq 0$$

$$-x^2 \neq -1$$

$$x^2 \neq 1$$

$$x \neq \pm 1$$

$$D = \mathbb{R} \setminus \{\pm 1\} = \{x \mid x \neq \pm 1\} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$$

Derivatives

$$\frac{d}{dx}\left[e^x\right] = e^x \quad , \quad \frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x}$$

Keys to more general derivatives.

e^x & $\ln(x)$ are inverses

$$3 = e^{\ln(3)} \Longrightarrow \frac{d}{dx}\left[3^x\right] = e^{\ln(3^x)} = e^{\ln(3)\cdot x} = e^{\ln(3)\cdot x}\cdot \ln(3)$$

$$\ln(e^3) = 3$$

$$\frac{d}{dx}\left[e^{u(x)}\right] = e^{u(x)}\cdot u'(x)$$

$$= \frac{d\, e^{u(x)}}{d(u(x))}\cdot\frac{du(x)}{dx}$$

$$\frac{d}{dx}\left[3^{\sin(x)}\right] = \ln(3)\cdot 3^{\sin(x)}\cdot\cos(x)$$

$$\boxed{\frac{d}{dx}\left[b^x\right] = \ln(b)\cdot b^x}$$

Derivatives of Log functions.

Change-of-base

$$\frac{d}{dx}\left[\log_b(x)\right] = \frac{1}{\ln(b)} \cdot \frac{1}{x}$$

$$\frac{d}{dx}\left[\log_b(u(x))\right] = \frac{1}{\ln(b)} \cdot \frac{1}{u(x)} \cdot u'(x)$$ Chain Version

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"my" way: Remember $\boxed{\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x}}$

$\boxed{\log_b(x) = \frac{\ln(x)}{\ln(b)}}$ → change of base.

$$\frac{d}{dx}\left[\log_3(x)\right] = \frac{1}{\ln(3)} \cdot \frac{1}{x}$$

$$\log_3(x) = \frac{\ln(x)}{\ln(3)} = \frac{1}{\ln(3)} \cdot \ln(x) \Longrightarrow$$

$$\frac{d}{dx}\left[\log_3(x)\right] = \frac{1}{\ln(3)} \frac{d}{dx}\left[\ln(x)\right] = \frac{1}{\ln(3)} \cdot \frac{1}{x}$$