About x-axis

$$\pi \int_a^b f(x)^2 \, dx$$

About y-axis

$$\pi \int_c^d g(y)^2 \, dy$$

2. Consider the region bounded by $y=(x-1)^3$, $y=0$, $x=1$, and $x=2$.

   a. (5 pts) Sketch this region and find its area.

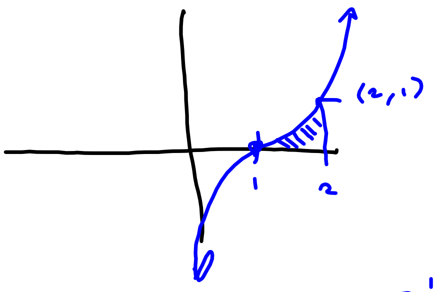
   b. (5 pts) Sketch the solid obtained by rotating this region about the line $y=3$. Include a representative washer on your graph and write the integral for finding its volume by the disk method.

   c. (5 pts) Evaluate the integral from part b.

   d. (5 pts) Sketch the solid obtained by rotating this region about the line $x=-3$. Include a representative washer on your graph and write the integral for finding its volume by the washer method.

   e. (5 pts) Evaluate the integral from part d.

(a) $y=(x-1)^3$

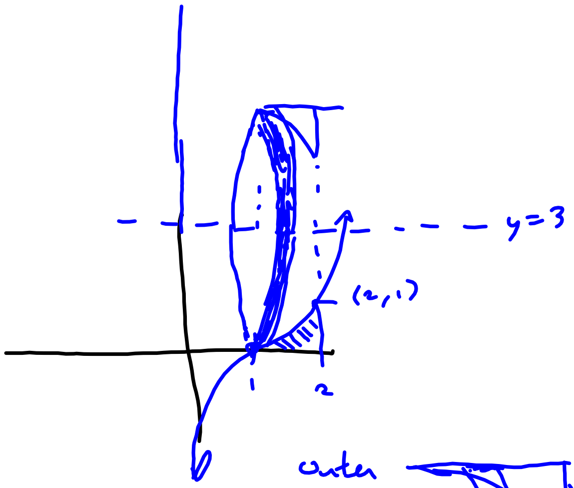


$\text{Area} = \int_1^2 (x-1)^3\,dx = \int_0^1 u^3\,du = \frac{u^4}{4}\Big]_0^1 = \frac{1^4}{4} = \frac{1}{4} = \text{Area}$

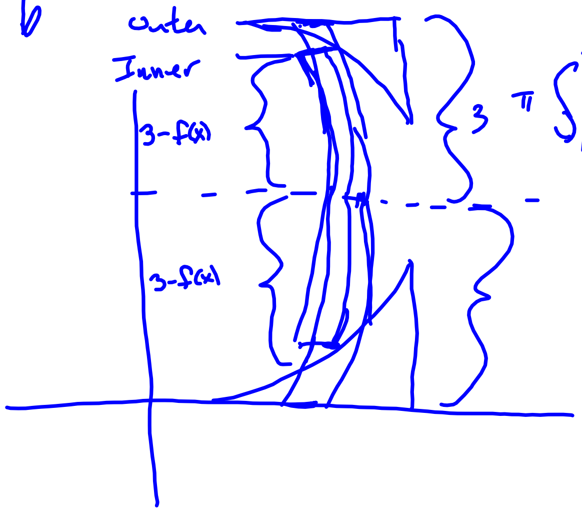
$u = x-1$ $\quad x=1 \Rightarrow u=1-1=0$

$du = dx$ $\quad x=2 \Rightarrow u=2-1=1$

(b) $y=(x-1)^3$



Terrible rendering



$\pi \int_1^2 \left(3^2 - (3-(x-1)^3)^2\right) dx$

ⓒ

$$\pi \int_1^2 \left(3^2 - (3-(x-1)^3)^2\right) dx$$

$$= \pi \int_1^2 3^2 dx - \pi \int_1^2 \left(9 - 2(3)(x-1)^3 + ((x-1)^3)^2\right) dx$$

$$= \boxed{\pi \left[9x\right]_1^2 - \pi \int_1^2 9\, dx \quad \text{cancels.}} + \pi \int_1^2 6(x-1)^3 dx - \pi \int_1^2 (x-1)^6 dx$$

$$+6\pi \left[\frac{(x-1)^4}{4}\right]_1^2 - \frac{\pi}{7}\left[(x-1)^7\right]_1^2$$

$$= 6\pi\left[\frac{1^4}{4} - \frac{0^4}{4}\right] - \frac{\pi}{7}\left[1^7 - 0^7\right] = 6\pi\left[\frac{1}{4}\right] - \frac{\pi}{7}\left[1\right]$$

$$= \frac{6\pi}{4} - \frac{\pi}{7} = \frac{3\pi}{2}\cdot\frac{7}{7} - \frac{\pi}{7}\cdot\frac{2}{2}$$

$$= \frac{21\pi - 2\pi}{14} = \boxed{\frac{19\pi}{14} = \text{Vol.}}$$

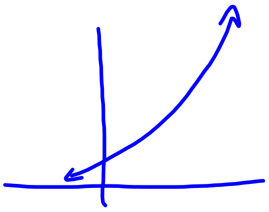
Chapter 6 Stuff (Review)

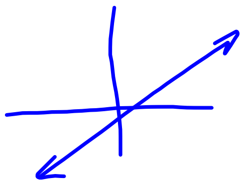
Show that $\frac{1}{x-2}$ is a 1-to-1 function

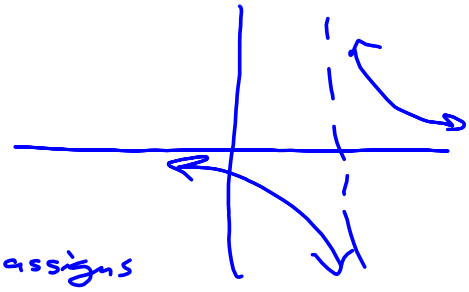
Assume $\frac{1}{x_1-2} = \frac{1}{x_2-2} \Longrightarrow$

$x_2 - 2 = x_1 - 2$

$x_2 = x_1 \Longrightarrow \frac{1}{x-2}$ is 1-to-1

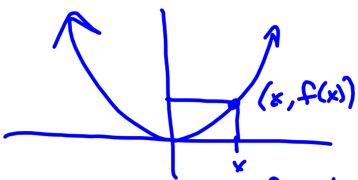






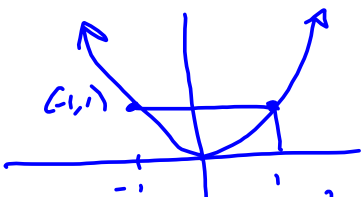
Function is a rule that assigns to each x in a domain to exactly one y in a range.

1-to-1 Function is a function such that each y in the range is assigned to exactly one x in the domain



x^2 is a function
Vertical line test

Not 1-to-1



Fails horizontal line test.
y=1 has x=-1 and x=1 in its "pre-image"

Suppose $x_1^2 = x_2^2$

$\sqrt{x_1^2} = \sqrt{x_2^2}$

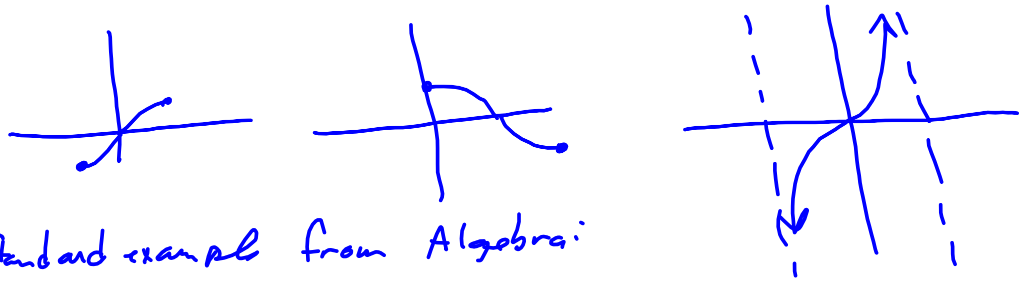
$|x_1| = |x_2|$

$x_1 = \pm x_2$!

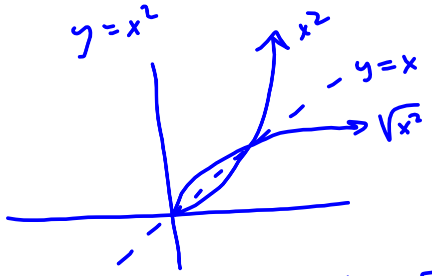
So 2 possibilities

Not 1-to-1.

This is why we restrict the domain of sine to $[-\frac{\pi}{2},\frac{\pi}{2}]$, cosine to $[0,\pi]$, and tangent to $(-\frac{\pi}{2},\frac{\pi}{2})$



Standard example from Algebra:



$f(x)$ & $f^{-1}(x)$ are reflections of one another about the line $y=x$.

Restrict Domain to $[0,\infty)$

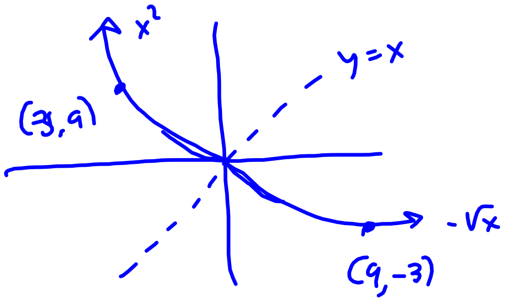
$y=x^2 \longrightarrow \sqrt{y} = \sqrt{x^2} = |x| \longrightarrow$

$|x| = \sqrt{y}$

$x = \pm\sqrt{y}$

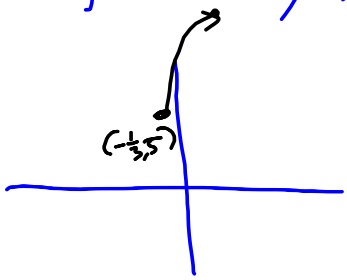
$y=\sqrt{x}$ is the inverse function.

$y=-\sqrt{x}$ " " " " if we restrict the domain to $(-\infty,0]$



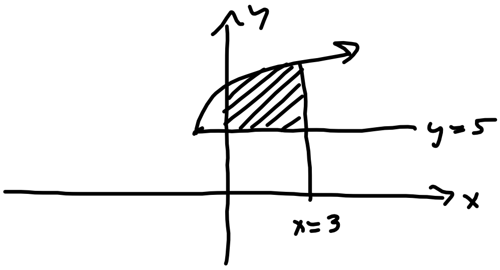
revolve $\sqrt{3x+1}+5$ about the y-axis

Region bdd by y-axis, $x=3$, $y=5$, $\sqrt{3x+1}+5$



$\sqrt{3x+1}+5$

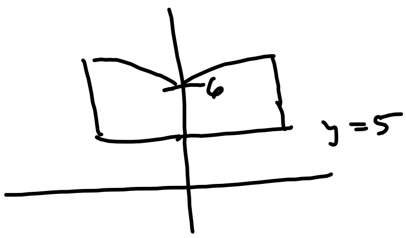
$\sqrt{3(x+\frac{1}{3})}+5$

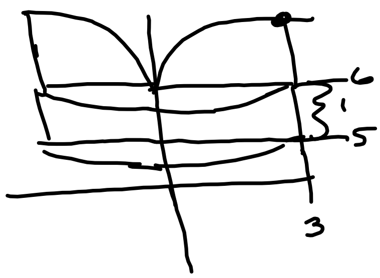


The volume of the solid of revolution

$(3, \sqrt{10}+5)$ $\sqrt{3(3)+1}+5$

$=\sqrt{10}+5$





$\cancel{\frac{1}{3}3x+1}$ $\sqrt{3x+1} + 5$

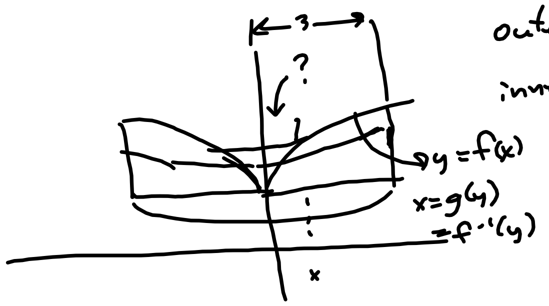
Lower cylinder:

$\pi\int_5^6 3^2 dx = \pi\left[9x\right]_5^6 = 9\pi\left[6-5\right] = 9\pi$

$= \sqrt{3(0)+1} + 5$

$= 1+5=6$

$\pi r^2 h = \pi(3)^2(1) = 9\pi$



outer = 3

inner = $x = g(y)$

$y = \sqrt{3x+1} + 5 \longrightarrow$

$x = \sqrt{3y+1} + 5 \longrightarrow$ probably shouldn't have swapped variables in this application

$x-5 = \sqrt{3y+1}$

$(x-5)^2 = 3y+1$ Just go from $y=f(x)$ to $x=g(y)$

$(x-5)^2 = 3y$

$\frac{(x-5)^2}{3} = y = f^{-1}(x)$

We want $x=g(y) = f^{-1}(y)$

$g(y) = \frac{(y-5)^2}{3}$

Vol = $\pi\int_6^{\sqrt{10}+5}\left(3^2 - \left(\left(\frac{y-5}{3}\right)^2\right)^2\right) dy$ + 9π from before

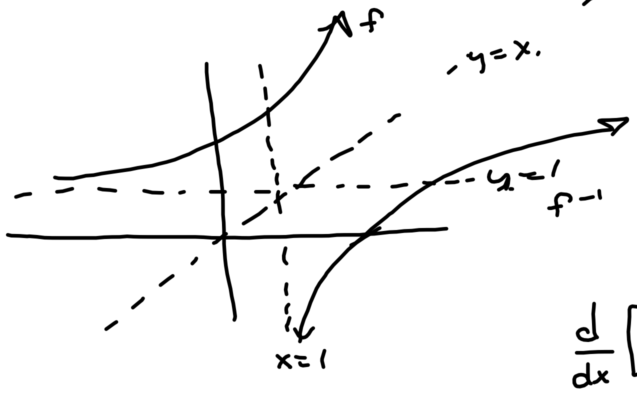
$y = e^{2x} + 1$

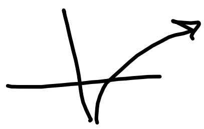
$y - 1 = e^{2x}$

$\ln(y-1) = \ln(e^{2x}) = 2x$

$\frac{1}{2}\ln(y-1) = x$

if $f(x) = e^{2x}+1$, then $f^{-1}(x) = \frac{1}{2}\ln(x-1)$





$\frac{d}{dx}\left[e^x\right] = e^x$

$\frac{d}{dx}\left[2^x\right] = \ln(2)\cdot 2^x$