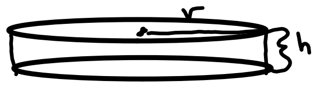
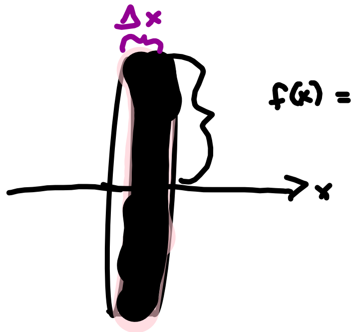
Volume of a right circular cylinder is area of the base times the height.

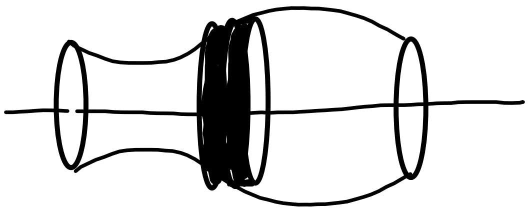


$V = \pi r^2 h$



$f(x)$ = radius

volume of this cylinder is $\pi f(x)^2 \Delta x$



$$V \approx \sum_{k=1}^{n} \pi f(x_k)^2 \Delta x$$

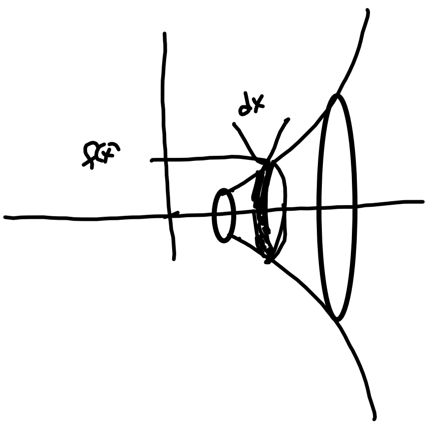
$$\approx \pi \sum_{k=1}^{n} f(x_k) \Delta x$$

$$\xrightarrow{\Delta x \to 0} \quad \text{OR} \quad \xrightarrow{n \to \infty} \quad \pi \int_a^b f(x)^2 \, dx$$

Revolving about the x-axis.

Likewise $\quad$ Volume $= \pi \int_c^d g(y)^2 \, dy$

Revolve $f(x) = x^2$ about the x-axis from $x = 1$ to $x = 3$



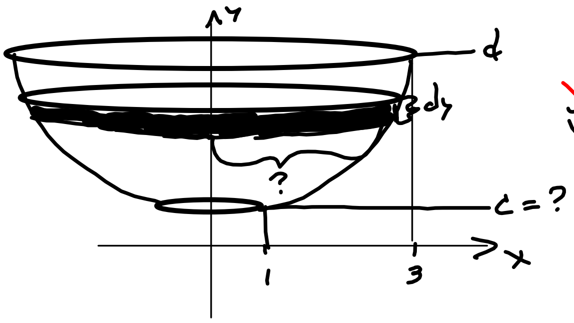
$$\text{Volume} = \pi \int_1^3 (x^2)^2 \, dx$$

$$= \pi \int_1^3 x^4 \, dx = \frac{\pi}{5} x^5 \Big]_1^3$$

$$= \frac{\pi}{5} \left[243 - 1 \right] =$$

$$= \frac{\pi}{5} \cdot 242 = \boxed{\frac{242\pi}{5}}$$

Revolve the same region about the *y*-axis.



Thx, Kallen

~~$y = x^4 \Rightarrow \sqrt[4]{y} = \sqrt[4]{x^4} = |x| = x$ since $x \geq 1$~~

~~$x = 3 \Rightarrow y =$~~

$$y = x^2 \Rightarrow \sqrt{y} = \sqrt{x^2} = |x| = x, \text{ since } x \geq 1 > 0$$

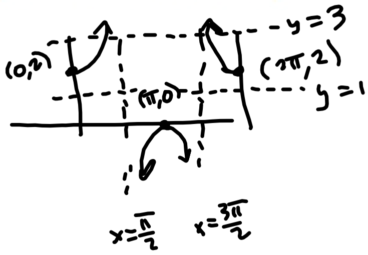
$$x = 1 \Rightarrow y = x^2 = 1$$

$$x = 3 \Rightarrow y = x^2 = 9$$

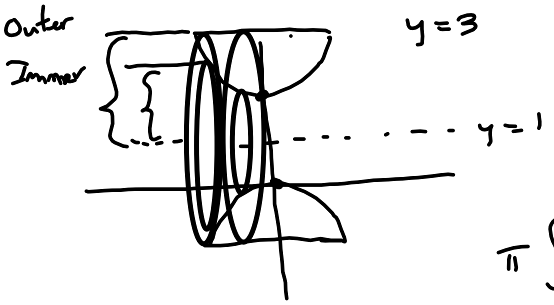
$$V = \pi \int_1^9 (\sqrt{y})^2 \, dy = \pi \int_1^9 y \, dy = \pi \left[\frac{y^2}{2} \right]_1^9 = \pi \left[\frac{81}{2} - \frac{1}{2} \right]$$

$$= \boxed{40\pi}$$

13. $y = 1 + \sec x$, $y = 3$; about $y = 1$, apparently from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$



Thx, Kory!

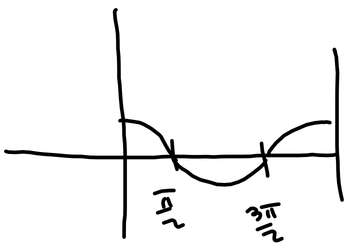


Outer = $3-1 = 2$

Inner = $1+\sec(x) - 1 = \sec(x)$

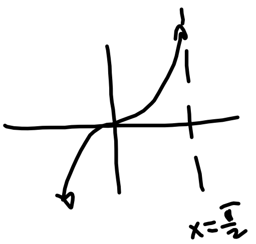
$$\pi \int_{-\frac{\pi}{2} \to -\frac{\pi}{3}}^{\frac{\pi}{2} \to \frac{\pi}{3}} \text{OUTER}^2\,dx - \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \text{Inner}^2\,dx$$

$$= \pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\text{outer}^2 - \text{inner}^2)\,dx$$



$$= 2\pi \int_0^{\frac{\pi}{2} \to \frac{\pi}{3}} (2^2 - \sec^2(x))\,dx$$

$$= 2\pi \Big[4x - \tan(x)\Big]_0^{\frac{\pi}{2} \to \frac{\pi}{3}}$$



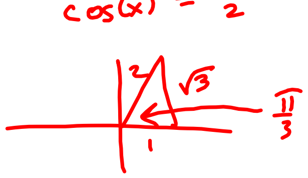
$$= 2\pi \left\{4\cdot\frac{\pi}{3} - \tan\left(\frac{\pi}{3}\right) - (0 - \tan(0))\right\}$$

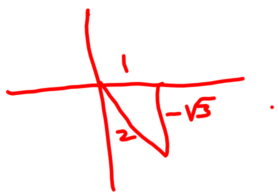
$$= 2\pi\left[\frac{4\pi}{3} - \sqrt{3}\right] = \frac{8\pi^2}{3} - 2\pi\sqrt{3}$$

Solve $\sec(x) + 1 = 3$! Left this step out.

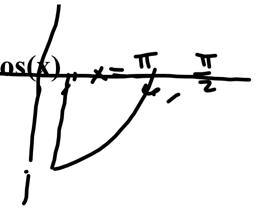
$\sec(x) = 2$

$\cos(x) = \frac{1}{2}$





Rotate $y = \cos(x)$, $x = \frac{\pi}{6}, \frac{\pi}{2}$ about the line $y = 10$

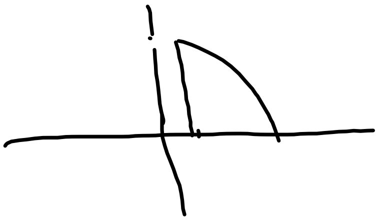


OUTER =

$y=10$

Outer $= 10$

inner $= 10 - \cos(x)$



$$\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(10^2 - (10-\cos(x))^2\right) dx$$

$$= \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (100 -$$

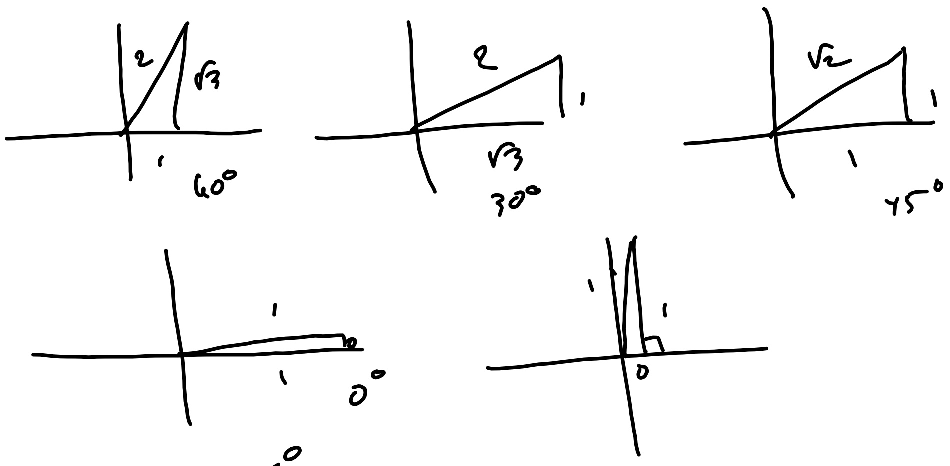
$$(10-\cos(x))^2 = (\cos(x)-10)^2$$

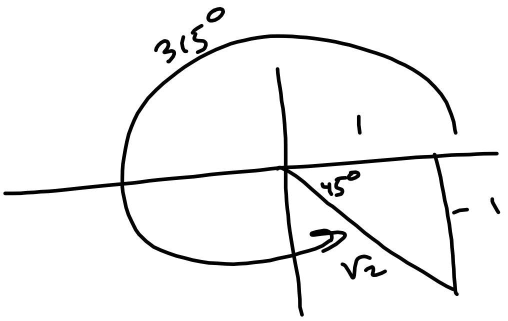
$$= \cos^2(x) - 20\cos(x) + 100$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2(x)\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1+\cos(2x)}{2}\,dx$$

Power-Reduction Formula from Trig:

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$



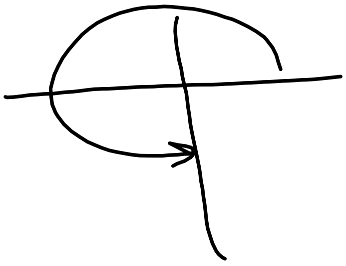


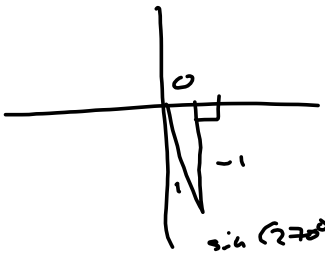
$\sin 315^\circ = -\frac{1}{\sqrt{2}}$

$\cos 315^\circ = \frac{1}{\sqrt{2}}$

$\tan 315^\circ = -1$

$\sin(270^\circ)$





$\sin(270^\circ) = -1$

$\cos(270^\circ) = 0$

$\tan(270^\circ) = \frac{-1}{0}$ $\nexists$