4.3 #19 Find the derivative of the function.

$$g(x) = \int_{3x}^{5x} \frac{u^2 - 2}{u^2 + 2}\, du$$

FTC I

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \qquad \int_a^{v(x)} f(t)\,dt = f(v(x))\,v'(x)$$

$$\sum_{k=1}^{n} f(k)\Delta x$$

$$\int_a^b f = \int_a^c f + \int_c^b f \quad , \text{for } c \in (a,b)$$

$$\int_{3x}^{5x} = \int_{3x}^{0} + \int_0^{5x} = -\int_0^{3x} + \int_0^{5x}$$

$$\frac{d}{dx}\int_{3x}^{5x} \frac{u^2-2}{u^2+2}\,du = \frac{d}{dx}\left[\int_{3x}^{0} \sim + \int_0^{5x} \sim\right]$$

$$= \frac{d}{dx}\left[-\int_0^{3x} \frac{u^2-2}{u^2+2}\,du + \int_0^{5x} \frac{u^2-2}{u^2+2}\,du\right]$$

$$= -\frac{d}{dx}\int_0^{3x} \frac{u^2-2}{u^2+2} + \frac{d}{dx}\int_0^{5x} \frac{u^2-2}{u^2+2}\,du = -\left(\frac{(3x)^2-2}{(3x)^2+2}\right)(3) + \left(\frac{(5x)^2-2}{(5x)^2+2}\right)(5)$$

$x^3 - y^3 = (x-3)(x^2+xy+y^2)$ Difference of cubes

$(x^3+y^3) = (x+3)(x^2-xy+y^2)$ Sum of cubes

Completing the square.

$x^2+bx = x^2+bx+\left(\frac{b}{2}\right)^2 - \frac{b^2}{a^2}$

$= \left(x+\frac{b}{2}\right)^2 - \frac{b^2}{a^2}$

$x^2+bx+c = \underbrace{x^2+bx+\left(\frac{b}{2}\right)^2} - \frac{b^2}{a^2} + \frac{c(a^2)}{(a^2)}$

$= \left(x+\frac{b}{2}\right)^2 + \frac{a^2c-b^2}{a^2}$

$x^2-bx+c = x^2-bx+\left(\frac{b}{2}\right)^2 - \frac{b^2}{a^2} + c^2$

$= \left(x-\frac{b}{2}\right)^2 + \frac{a^2c-b^2}{a^2}$

Scratch

$= \frac{a^2c-b^2}{a^2}$

$$\int \frac{dx}{\sqrt{x^2+2x-1}}$$

$x^2+2x-1 = x^2+2x+1^2-1^2$

$= (x+1)^2-2$

$$= \int \frac{dx}{\sqrt{(x+1)^2-2}}$$

Let $u = x+1$ $\Rightarrow$ $\int \frac{du}{\sqrt{u^2-2}} = \ln\left|(x+1) + \sqrt{(x+1)^2-2}\right| + C$

$du = dx$

$(\sqrt{2})^2$

43. $\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u+\sqrt{u^2-a^2}\right| + C$

Kahn Academy

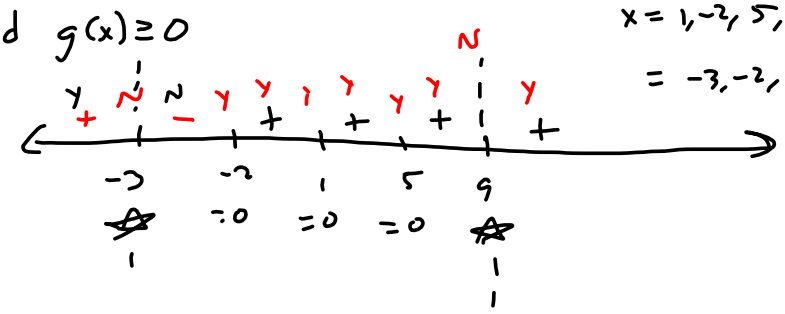
Sign Pattern

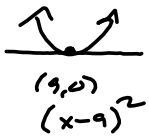
What's the domain for $f(x)=\sqrt{g(x)} = \sqrt{\frac{(x-1)^2(x+2)^3(x-5)^4}{(x+3)(x-9)^2}}$

$x = 1, -2, 5, -3, 9$

$= -3, -2, 1, 5, 9$

Need $g(x) \geq 0$





Need $g(x) \geq 0$ $D(f) = (-\infty, -3) \cup [-2, 9) \cup (9, \infty)$