$$\frac{d}{dx}\left[\sqrt[5]{(x^2-5x)^3}\right] = \frac{d}{dx}\left[(x^2-5x)^{\frac{3}{5}}\right] = \frac{3}{5}(x^2-5x)^{-\frac{2}{5}}(2x-5)$$

$$\int (x^2-5x)^{-\frac{2}{5}}(2x-5)\,dx = \int u^{-2/5}\,du = \frac{u^{\frac{3}{5}}}{\frac{3}{5}} + C = \frac{5}{3}(u^{\frac{3}{5}})+C$$

$$\int u^{-\frac{2}{5}}du \qquad = \frac{5}{3}(x^2-5x)^{\frac{3}{5}}+C$$

$u = x^2-5x$

$du = (2x-5)dx$

$$\int \underbrace{\sin(x^2-5x)}\,\underbrace{(2x-5)dx}_{du} = \int \sin(u)\,du = -\cos(u)+C$$

$$= -\cos(x^2-5x)+C$$

$$\int \frac{\cos\left(\frac{\pi}{x^5}\right)}{x^6}\,dx = \int \cos\left(\frac{\pi}{x^5}\right)\left(\frac{1}{x^6}\right)dx$$

$u = \frac{\pi}{x^5} = \pi x^{-5}$

$\Longrightarrow du = -5\pi x^{-6}\,dx = \frac{5\pi}{x^6}dx$

$$= \frac{-1}{5\pi}\int \cos\left(\frac{\pi}{x^5}\right)\left(\frac{-5\pi}{x^6}dx\right) = \frac{-1}{5\pi}\int \cos(u)\,du$$

$$= \frac{-1}{5\pi}\left[\sin(u)\right]+C = \frac{-1}{5\pi}\left[\sin\left(\frac{\pi}{x^5}\right)\right]+C$$

$u = \frac{\pi}{x^5} = \pi x^{-5} \longrightarrow du = -5\pi x^{-6}dx \longrightarrow$

$dx = \frac{du}{-5\pi x^{-6}} = -\frac{1}{5\pi}x^6\,du \longrightarrow$

$$\int \frac{\cos\left(\frac{\pi}{x^5}\right)}{\cancel{x^6}}\cdot\left(-\frac{1}{5\pi}\cancel{x^6}\right)du$$

$$= -\frac{1}{5\pi}\int \cos(u)\,du$$

§ 4.5 #10

$$\int x(5x-7)^6\,dx = \frac{1}{5}\int x\,u^6\cdot 5\,dx = \frac{1}{5}\int x\,u^6\,du$$

$u = 5x-7 \Longrightarrow u+7 = 5x \Longrightarrow \frac{u+7}{5} = x$

$du = 5\,dx$

$$= \frac{1}{5}\int\left(\frac{u+7}{5}\right)u^6\,du = \frac{1}{25}\int (u+7)\,u^6\,du$$

$$= \frac{1}{25}\int\left(u^7 + 7u^6\right)du = \frac{1}{25}\left[\frac{u^8}{8} + \frac{7u^7}{7}\right] + C$$

$$= \frac{1}{25}\left[\frac{(5x-7)^8}{8} + (5x-7)^7\right] + C$$

$$\int x^3\sqrt{x^2+29}\,dx$$

$$\left(\begin{array}{l} u = x^2+29 \Rightarrow u-29 = x^2 \\ du = 2x\,dx \end{array}\right) \qquad \sqrt{u} = u^{\frac{1}{2}}$$

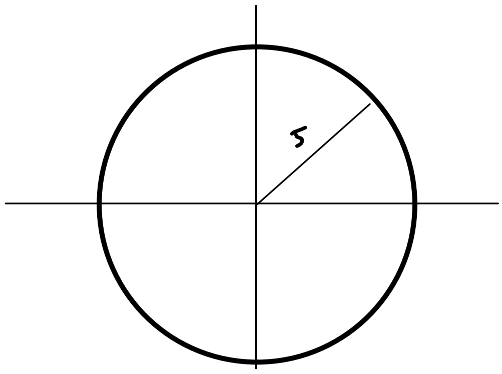
$$= \frac{1}{2}\int x^2\sqrt{x^2+29}\cdot 2x\,dx = \frac{1}{2}\int (u-29)\sqrt{u}\cdot du$$

$$= \quad = \frac{1}{2}\int \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right)du \text{, etc.}$$

$$\int \sec^4(x)\tan(x)\,dx = \int \underbrace{\sec^3(x)}_{u^n}\,\underbrace{\sec(x)\tan(x)\,dx}_{du}$$

$$\left(\begin{array}{l} u = \sec(x) \\ du = \sec(x)\tan(x)\,dx \end{array}\right)$$

$$= \int u^3\,du = \frac{u^4}{4} + C = \boxed{\frac{\sec^4(x)}{4} + C}$$



$x^2+y^2=5^2$

$x^2+y^2=25$

$y^2=25-x^2$

$y=\pm\sqrt{25-x^2}$

$y=\sqrt{25-x^2}$ top half

$y=-\sqrt{25-x^2}$ bottom half.

$$\int_{-5}^{5}\sqrt{25-x^2}\,dx = \tfrac{1}{2} \text{ the area of circle of radius } r=5!$$

$$\tfrac{1}{2}(\pi\cdot 5^2) = \tfrac{25}{2}\pi\,!$$

$u = 25 - x^2$

$du = -2x\,dx$, but there's no "x" in the integrand.

$$\int_{-5}^{5} x\sqrt{25-x^2}\,dx = -\tfrac{1}{2}\int (25-x^2)^{\frac{1}{2}}(-2x\,dx)$$

$u = 25 - x^2$

$du = -2x\,dx$

$x\sqrt{25-x^2} = f(x)$

$f(-x) = (-x)\sqrt{25-(-x)^2}$

$= -x\sqrt{25-x^2} = -f(x)\,!$

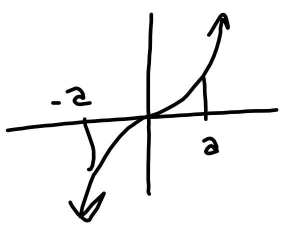
$\int_{\text{lower limit}}^{\text{upper limit}} \text{integrand}$

∞

$$\int_{-5}^{5} x\sqrt{25-x^2}\,dx$$

$f(x)=x^2$ is even, b/c $f(-x)=(-x)^2=(-x)(-x)=+x^2=f(x)=f(-x)$

$g(x)=x^3$ is odd, b/c $f(-x)=(-x)^3=-x^3=-f(x)$



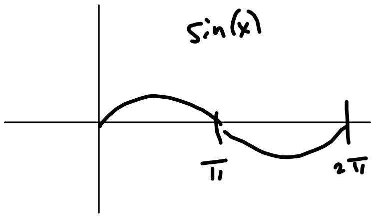
positive area on right
cancels negative area on left

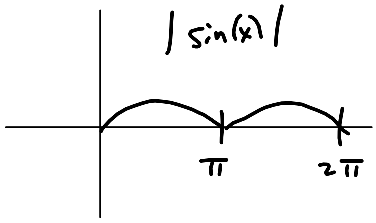
Net Change versus Total Change.

$$\int_a^b f(x)\,dx \quad \text{-vs-} \quad \int_a^b |f(x)|\,dx$$

$$\int_0^{2\pi} \sin(x)\,dx = 0 \quad , \text{but} \quad \int_0^{2\pi} |\sin(x)|\,dx$$

$$|\sin(x)| = \begin{cases} \sin(x) & \text{if } \sin(x) \geq 0 \\ -\sin(x) & \text{if } \sin(x) < 0 \end{cases}$$





$$|\sin(x)| = \begin{cases} \sin(x) & \text{if } 0 \leq x \leq \pi \\ -\sin(x) & \text{if } \pi < x \leq 2\pi \end{cases}$$

$$\int_0^{2\pi} |\sin(x)|\,dx = \int_0^{\pi} \sin(x)\,dx + \int_{\pi}^{2\pi} -\sin(x)\,dx$$

$$= \int_0^{\pi} \sin(x)\,dx - \int_{\pi}^{2\pi} \sin(x)\,dx = \Big[-\cos(x)\Big]_0^{\pi} - \Big[-\cos(x)\Big]_{\pi}^{2\pi}$$

$$= -\cos(\pi) - (-\cos(0)) - \Big[-\cos(2\pi) - (-\cos(\pi))\Big]$$

$$= -(-1) - (-1) - \Big[-1 - (-(-1))\Big]$$

$$= 1 + 1 - \Big[-1 - 1\Big] = 2 - (-2) = 4$$