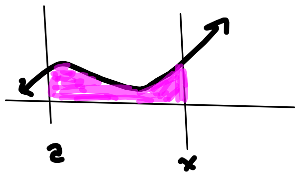
Let $g(x) = \int_a^x f(x)\,dx$.

Visually, $g(x)$ is $\overbrace{\text{the area}}^{\text{SIGNED}}$ under $f(x)$ from a to x. If $f(x) \geq 0$, it's the positive area under $f(x)$ over $[a,b]$. It makes sense to think of $\int_a^x f(x)\,dx$ as an increasing function of x if $f(x) \geq 0$ on $[a,b]$

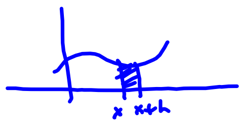
$\int_a^x f(x)\,dx$ = shaded area:



Let $a < b < c$

Then $\int_a^c = \int_a^b + \int_b^c$ and so $\int_a^c - \int_a^b = \int_b^c$

$\int_a^{x+h} - \int_a^x = \int_x^{x+h}$.



h is usually quite small.

$g(x) = \int_a^x f(x)\,dx \longrightarrow g(x+h) - g(x) =$

$\int_a^{x+h} - \int_a^x = \int_x^{x+h}$. Hence

$$\frac{g(x+h)-g(x)}{h} = \frac{1}{h}\left(g(x+h)-g(x)\right) = \frac{1}{h}\int_x^{x+h} f(x)\,dx$$

By E.V.T., $\exists\ u,v \in [x, x+h] \ni$ $f(u) = \min\limits_{v\in[x,x+h]} \{f(x)\}$ and $f(v) = \max\limits_{x\in[x,x+h]}\{f(x)\}$, so that upper & lower estimates on $\int_x^{x+h} f(x)\,dx$ are

$$f(u)\underbrace{(x+h-x)}_{b-a} \le \int_x^{x+h} f(x)\,dx \le f(v)h \implies$$

$$f(u) \le \frac{1}{h}\int_x^{x+h} f(x)\,dx \le f(v)$$

$$\downarrow h\to 0 \qquad\qquad \downarrow h\to 0$$

$$f(x) \le \frac{d}{dx}\int_a^x f(x)\,dx \le f(x)$$

i.e. $\frac{d}{dx}\int_a^x f(x)\,dx = f(x)$ Fundamental Theorem of Calculus Part I

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FTC II

Let $f(x)$ be cnt's on $[a,b]$ and $\ni$ $F'(x)=f(x)$

Then $\int_a^b f(x)\,dx = F(b)-F(a)$.

Pf

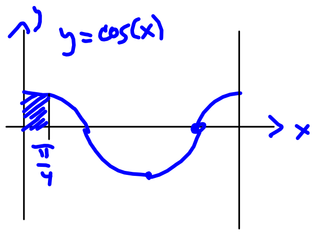
Define $g(x)=\int_a^x f(x)\,dx$. $g(x)$ is an antiderivative of $f(x)$ and so $F(x)=g(x)+C$ for some $C\in\mathbb{R}$, and $\forall x\in(a,b)$. check endpoints

$$\left.\begin{aligned} g(a)=0 &= \int_a^a f(x)\,dx = F(a)+C \\ g(b) &= \int_a^b f(x)\,dx = F(b)+C \end{aligned}\right\}\rightarrow$$

$$F(b)-F(a) = g(b)-C-(g(a)-C) = g(b)-g(a)$$

$$= g(b) = \int_a^b f(x)\,dx \quad \blacksquare$$

$$\int_0^{\frac{\pi}{4}}\cos(x)\,dx = \sin(x)\Big]_0^{\pi/4} = \sin\left(\tfrac{\pi}{4}\right)-\sin(0) = \frac{1}{\sqrt{2}}-0 = \boxed{\frac{1}{\sqrt{2}}}$$



$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Big]_0^1 = \frac{1}{3}$$

$$a+k\left(\frac{b-a}{n}\right) = x_k = 0+k\left(\frac{1}{n}\right)$$

$$\frac{1}{n}\sum_{k=1}^n f(x_k) = \frac{1}{n}\sum_{k=1}^n\left(\frac{k}{n}\right)^2 = \frac{1}{n}\cdot\frac{1}{n^2}\sum_{k=1}^n k^2 = \frac{1}{n^3}\left[\frac{n^3+\cdots}{3}\right] \xrightarrow{n\to\infty} \frac{\cancel{n^3}+\underbrace{\cdots}_{\to 0}}{3\cancel{n^3}}$$

$$\xrightarrow{n\to\infty}\frac{1}{3}$$

The Chain Rule Version of FTC I

$$\frac{d}{dx}\left[g(h(x))\right] = \frac{dg}{dh}\cdot\frac{dh}{dx}$$

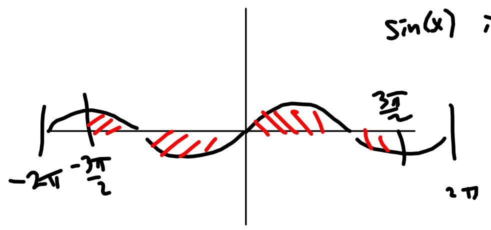
$$\frac{d}{dx}\left[\sin(x^2+3x)\right] = \cos(x^2+3x)\cdot(2x+3)$$

$$\frac{d}{dx}\left[\int_0^x \sin(w^2+3w)\,dw\right] = \sin(x^2+3x)$$

$$\frac{d}{dx}\left[\int_0^{\cos(x)} \sin(t^2+3t)\,dt\right] = \sin\left(\cos^2(x)+3\cos(x)\right)\cdot\left(-\sin(x)\right)$$

$$\frac{d}{dx}\left[\int_0^{x^2+3x} \sin(t)\,dt\right] = \sin(x^2+3x)\cdot(2x+3)$$

sin(x) is odd

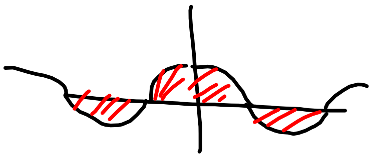
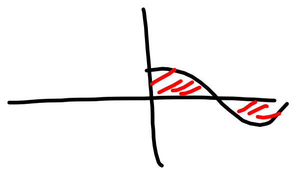


$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} \text{ODD}\,dx = 0$$

The definite integral of an odd function over a symmetric interval is zero.

The definite integral of an EVEN function over a symmetric interval:

$$\int_{-5}^{5}\frac{\cos(x)}{x^2+5}\,dx = 2\int_0^5 \frac{\cos(x)}{x^2+5}\,dx$$

 $= 2$ 

The number of umbrellas plus the number of raincoats is 50.

Let x = the # of umbrellas
y = " " " raincoats. Then

$$x + y = 50$$