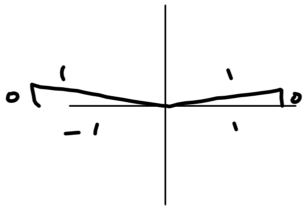
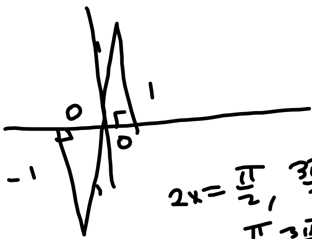
$f(x) = 4\sin^2(2x) - 1 = 4[\sin(2x)]^2 - 1$

$f'(x) = 8\sin(2x)\cdot\cos(2x)\cdot 2 = 16\sin(2x)\cos(2x) \overset{SET}{=} 0$

$\Rightarrow \sin(2x) = 0$ or $\cos(2x) = 0$





$2x = 0, \pi, 2\pi, 4\pi$

$x = 0, \frac{\pi}{2}, \pi, 2\pi$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

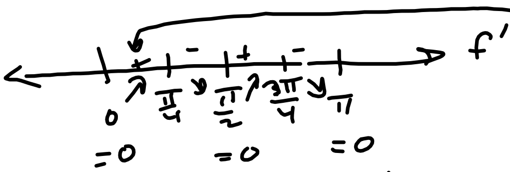
$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

What's the period of $\sin(x)$?

$\sin(2x)$

Want $2x = 2\pi$

$\Rightarrow x = \pi =$ Period



Test $16\sin(2x)\cos(2x)$

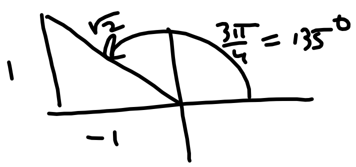
@ $x = \frac{\pi}{4}, \frac{3\pi}{4}$

$$\cancel{f'\left(\tfrac{\pi}{4}\right) = 16\sin\left(2\left(\tfrac{\pi}{4}\right)\right)\left(\cos\left(2\left(\tfrac{\pi}{4}\right)\right)\right) = 16\sin\left(\tfrac{\pi}{2}\right)\cos\left(\tfrac{\pi}{2}\right)}$$

This is where I realized I'd only found 1/2 of the zeros of the derivative. Didn't work the cos(2*x*) half, at first.

$$f'\left(\tfrac{\pi}{8}\right) = 16\sin\left(2\left(\tfrac{\pi}{8}\right)\right)\cos\left(2\left(\tfrac{\pi}{8}\right)\right)$$
$$= 16\left(\sin\left(\tfrac{\pi}{4}\right)\right)\left(\cos\left(\tfrac{\pi}{4}\right)\right)$$
$$= 16\left(\tfrac{1}{\sqrt{2}}\right)\left(\tfrac{1}{\sqrt{2}}\right) = 16\cdot\tfrac{1}{2} = 8 > 0 \quad +$$

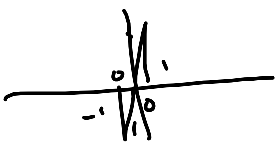
$$f'\left(\tfrac{3\pi}{8}\right) = 16\sin\left(\tfrac{3\pi}{4}\right)\cos\left(\tfrac{3\pi}{4}\right) = 16\left(\tfrac{1}{\sqrt{2}}\right)\left(-\tfrac{1}{\sqrt{2}}\right) = -8 < 0$$

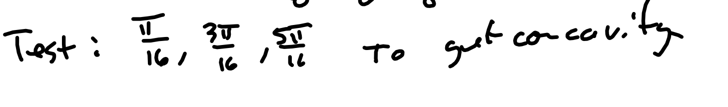


STILL Need $f''(x)$

$$f'(x) = 16\sin(2x)\cos(2x) = 8\cdot\left(2\sin(2x)\cos(2x)\right)$$
$$= 8\left(\sin(4x)\right)$$

$$\Rightarrow f''(x) = 32\cos(4x) \overset{\text{SET}}{=} 0 \longrightarrow$$
$$\cos(4x) = 0 \longrightarrow 4x = \tfrac{\pi}{2}, \tfrac{3\pi}{2}, \tfrac{5\pi}{2}$$
$$x = \tfrac{\pi}{8}, \tfrac{3\pi}{8}, \tfrac{5\pi}{8}, \tfrac{7\pi}{8}$$



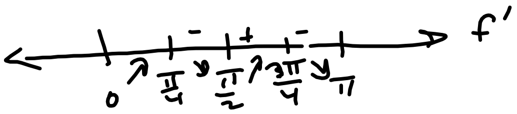


Test: $\frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}$ To get concavity

$f(x) = 4\sin^2(2x) - 1 = 4[\sin(2x)]^2 - 1$ **Has period Pi.**

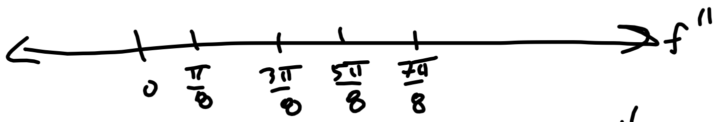
$f'(x) = 8\sin(2x)\cdot\cos(2x)\cdot 2 = 16\sin(2x)\cos(2x) \overset{SET}{=} 0$

$= 8\sin(4x)$



$\Rightarrow f''(x) = 32\cos(4x) \overset{SET}{=} 0 \longrightarrow 4x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$



Test: $\frac{\pi}{16}, \frac{3\pi}{16}, \frac{5\pi}{16}$ to get concavity

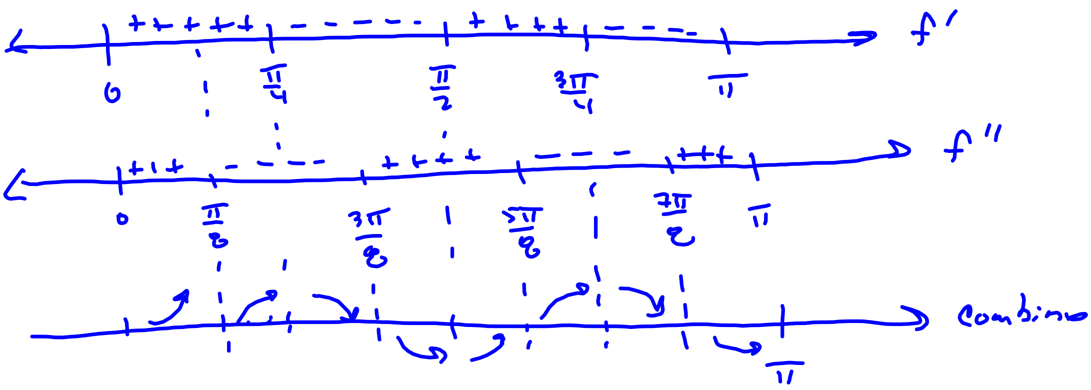
$f''\left(\frac{\pi}{16}\right) \approx$ 22.62741700

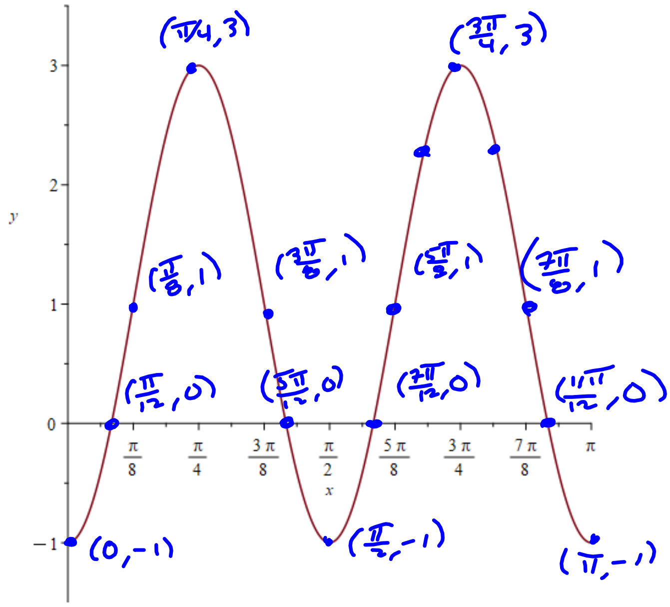
$f''\left(\frac{3\pi}{16}\right) \approx$ -22.62741700

$f''\left(\frac{7\pi}{16}\right) \approx$ 22.62741700

$f''\left(\frac{11\pi}{16}\right) \approx$ -22.62741700

$f''\left(\frac{15\pi}{16}\right) \approx$ 22.62741700



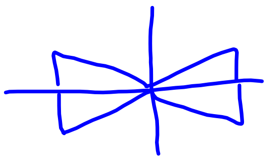


Don't confuse your test values with inflection points, like I did.

$$4\sin^2(2x) - 1 = 0$$

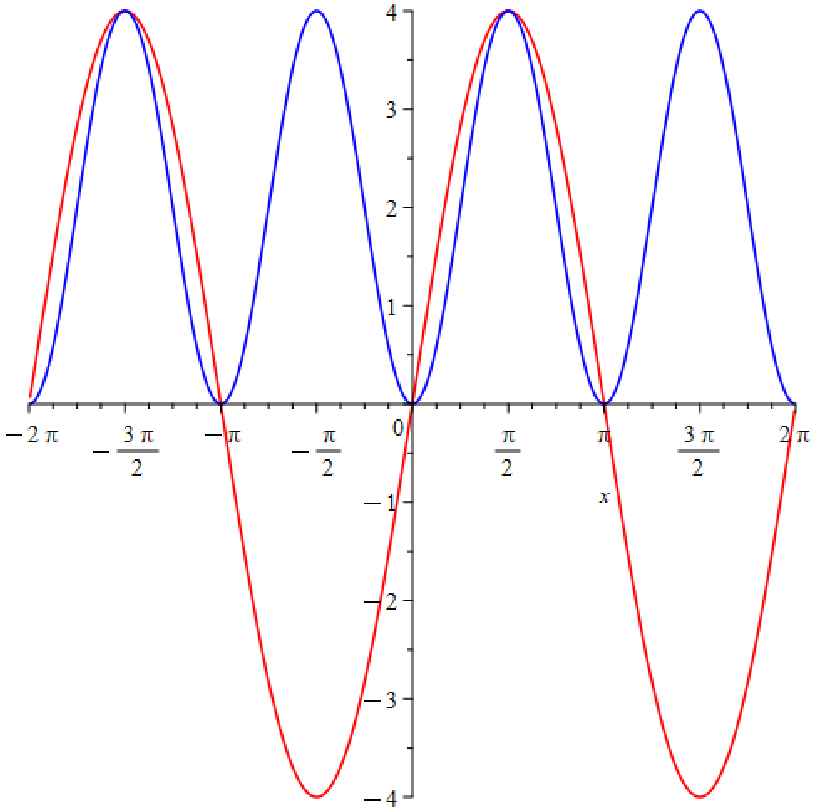
$$\sin^2(2x) = \frac{1}{4}$$

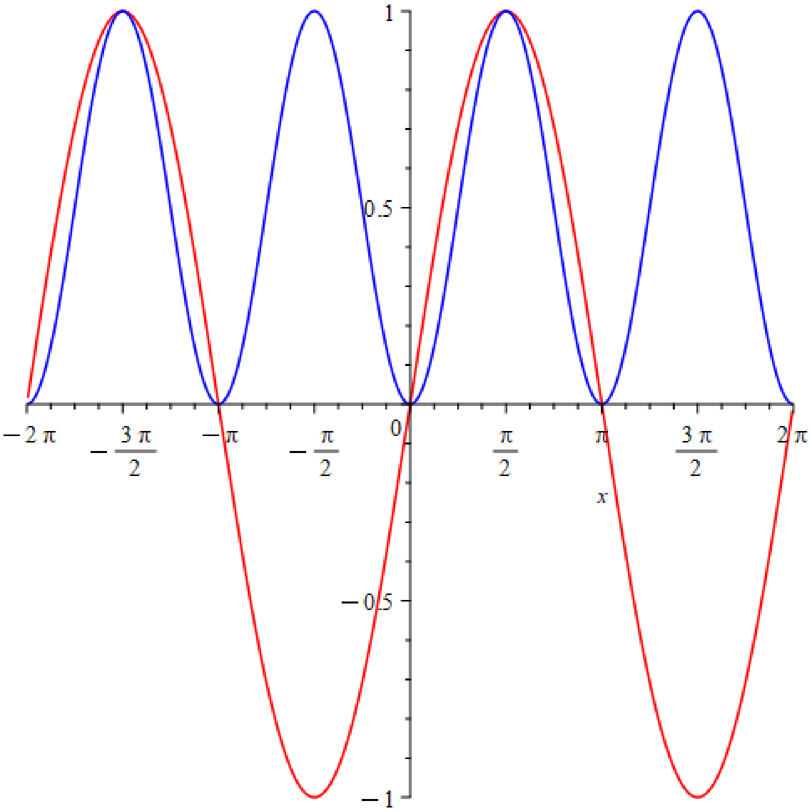
$$\sin(2x) = \pm\frac{1}{2}$$

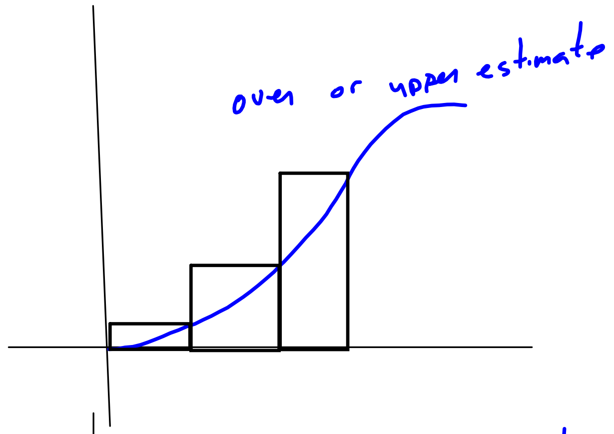


$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6},$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$$


4
3
2
1
0
− 1
− 2
− 3
− 4
− 2 π
− 3 π / 2
− π
− π / 2
π / 2
π
3 π / 2
2 π
x

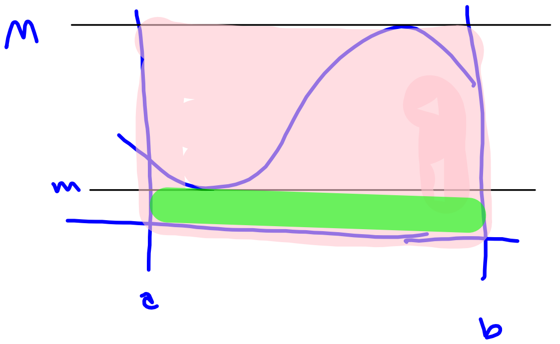

1
0.5
0
−0.5
−1
−2 π
− 3 π / 2
− π
− π / 2
π / 2
π
3 π / 2
2 π
x



Lower Estimate $\le$ Upper Estimate

A Lower Bound on $\int_a^b f(x)\,dx = m(b-a)$, where

$m = \min\limits_{x\in[a,b]}\{f(x)\}$ $\quad$ $M = \max\limits_{x\in[a,b]}\{f(x)\}$



$= m(b-a)$

$= M(b-a)$

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

Equal-width:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=1}^{n} f\left(a + \frac{b-a}{n}k\right) = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{b-a}{n}\right) f\left(a + \frac{b-a}{n}k\right)$$

In general

$$\int_a^b f(x)\,dx = \lim_{|\Delta x_k| \to 0} \sum_{k=1}^{n} f(a + \Delta x_k)\,\Delta x_k$$

$\max_{k\in\mathbb{N}} \{\Delta x_k\}$ = "mesh of the partition"