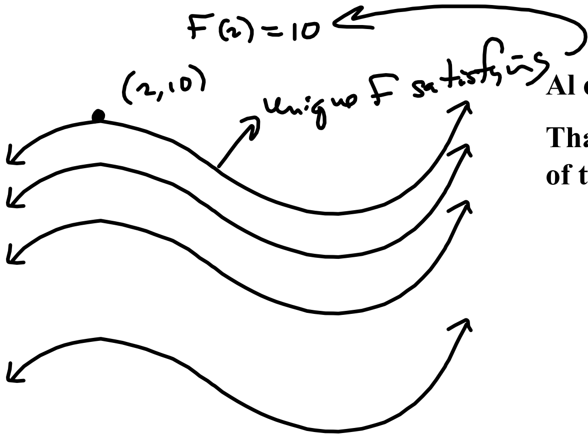
Section 3.9

Definition - A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.



Al of these have the same derivative.

That means they're all an antiderivative of the same function.

$F(x) = x^3 - 2x^2 + 5x - 7$ is an antiderive for

$f(x) = F'(x) = 3x^2 - 4x + 5$

so is $x^3 - 2x^2 + 5x$

so is $x^3 - 2x^2 + 5x + 7000$ } Have the same derivative.

$f(x) = \sin(x) \implies$

$F(x) = -\cos(x) + C$ for any $C \in \mathbb{R}$.

$\int f(x)\,dx = F(x) + C$

$\int \sin(x)\,dx = -\cos(x) + C$

$\int \cos(x) = \sin(x) + C$

$\int \tan(x)\,dx = \ln|\tan(x) + \sec(x)| + C$

(Won't be proved 'til we do exponentials & logs)

$\int \sec(x)\tan(x)\,dx = \sec(x) + C$

$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

Week 9 Written

$f''(x) = \sin(x)$, $f'(0) = 10$, $f(0) = -1$

What's $f(x)$?

$\Rightarrow f'(x) = -\cos(x) + C$

$f'(0) = 10 = -\cos(0) + C = -1 + C = 10 \Rightarrow$

$\boxed{C = 11} \Rightarrow$

$f'(x) = -\cos(x) + 11$

$\Rightarrow f(x) = -\sin(x) + 11x + D$

$f(0) = -1 = -\sin(0) + 11(0) + D = \boxed{D = -1}$

$\Rightarrow \boxed{f(x) = -\sin(x) + 11x - 1}$

Is the unique function satisfying all the conditions.

$$\frac{x^2-9}{x-5}$$

$$\begin{array}{r|rrr} 5 & 1 & 0 & -9 \\ & & 5 & 25 \\ \hline & 1 & 5 & 16 \end{array}$$

Says: DIVISION ALGORITHM

$$x^2-9 = (x-5)(x+5)+16$$

$$\frac{x^2-9}{x-5} = x+5+\frac{16}{x-5}$$

$$28 = 3\cdot 9 + 1$$

$$\frac{28}{3} = 9 + \frac{1}{3}$$

Dividend = Divisor · Quotient + Remainder

Find all solutions between 0 and 2Pi for the equation:

$4\sin^2(2x) - 1 = 0$

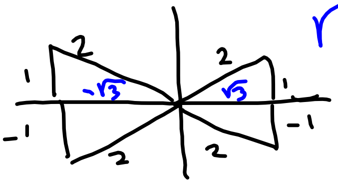
$4\sin^2(2x) = 1$

$\sin^2(2x) = \frac{1}{4}$

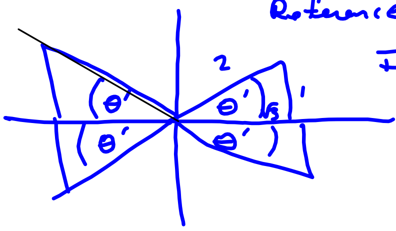
$\sin(2x) = \pm\frac{1}{2}$

want all $x \in [0, 2\pi)$

$\Rightarrow$ .. .. $2x \in [0, 4\pi)$



$\sqrt{2^2 - 1^2} = \sqrt{4-1} = \sqrt{3}$



Reference Angle θ'

In QI

$\arcsin(\frac{1}{2}) = 30^\circ = \theta'$

$\arcsin(\sin(2x)) = 30^\circ$

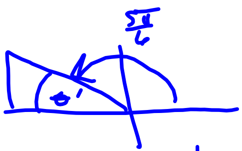
$2x = 30^\circ = \frac{\pi}{6}$

$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

$\Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

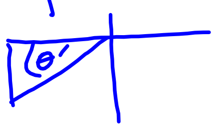
$\frac{\pi}{6} = \arcsin(\frac{1}{2})$

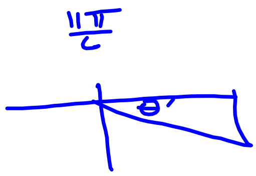
$\theta' = \arcsin(\frac{1}{2})$



$\frac{5\pi}{6} = \pi - \arcsin(\frac{1}{2})$

$\frac{7\pi}{6} = \pi + \arcsin(\frac{1}{2})$





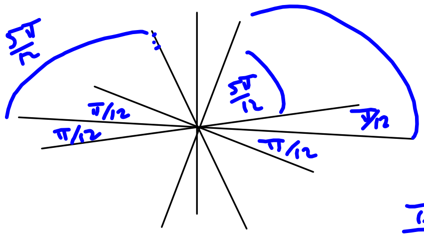
$\frac{11\pi}{6} = 2\pi - \arcsin\left(\frac{1}{2}\right)$

Find ALL solutions!

$\frac{5\pi}{12} = 75^\circ$

$\frac{\pi}{12} = 15^\circ$

$\frac{7\pi}{12} = 105^\circ$



$\frac{\pi}{12} + n\pi, \ \frac{5\pi}{12} + n\pi, \ \frac{7\pi}{12} + n\pi, \ \frac{11\pi}{12} + n\pi$

Sol'n set:

$\left\{ \frac{\pi}{12} + n\pi, \frac{5\pi}{12} + n\pi, \frac{7\pi}{12} + n\pi, \frac{11\pi}{12} + n\pi \;\middle|\; n \in \mathbb{Z} \right\}$

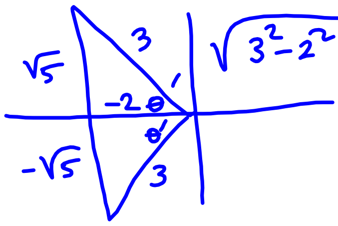
Let $A = \left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12} \right\}$

$x \in \{ y + n\pi \mid y \in A, n \in \mathbb{Z} \}$

Three plus five is eight

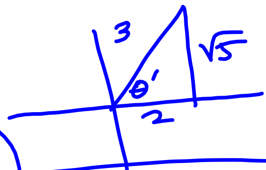
$3+5=8$

Draw Two triangles satisfying $\cos\theta = -\frac{2}{3}$



$\sqrt{3^2-2^2} = \sqrt{9-4} = \sqrt{5}$

Give exact sol'ns in $[0, 2\pi)$



$\boxed{\theta = \pi - \theta', \ \pi + \theta'}$ $\qquad$ $\boxed{\theta' = \arccos\left(\frac{2}{3}\right)}$

$\pi - \arccos\left(\frac{2}{3}\right), \ \pi + \arccos\left(\frac{2}{3}\right)$

$= \pi - \arcsin\left(\frac{\sqrt{5}}{3}\right), \ \pi + \arcsin\left(\frac{\sqrt{5}}{3}\right)$

$= \pi - \arctan\left(\frac{\sqrt{5}}{2}\right), \ \pi + \arctan\left(\frac{\sqrt{5}}{2}\right)$

Draw triangle with same reference angle in Quadrant I, and then use it!

Find ALL solutions to the equation.

$\pi - \theta' + 2\pi n, \ \pi + \theta' + 2\pi n, \ n \in \mathbb{Z}$

$\boxed{(2n+1)\pi \pm \theta', \ n \in \mathbb{Z}}$

$\pi + 2\pi n = \pi(1+2n) = (2n+1)\pi$

I will post the Trig Midterm and its solutions on the Trig page.