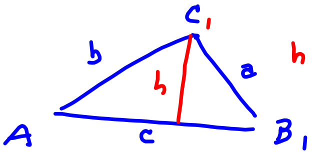
Questions?

Week 8 Written. #1 e.

#34 from Section 3.3

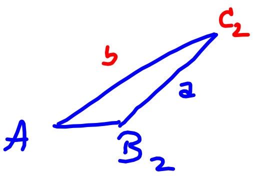
arcsine gives you angle B_1 = acute B

$180^\circ - B_1 = B_2$ = obtuse B.



$h < a < b$

B is acute

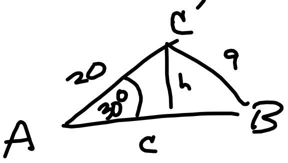


B is obtuse

The case where Law of Sines has 2 solutions

#1b

$A = 30^\circ,\ b = 20,\ c = ?,\ a = 9$



$\frac{h}{20} = \sin 30^\circ \longrightarrow$

$h = 20\sin 30^\circ = 20\left(\frac{1}{2}\right) = 10 < 20 = b$

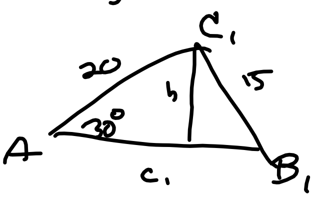
$\longrightarrow$ 2 sol'ns

$\frac{\sin B}{b} = \frac{\sin A}{a} \longrightarrow \frac{\sin B}{20} = \frac{\sin 30^\circ}{9} \longrightarrow \sin B = \frac{20 \sin 30^\circ}{9}$

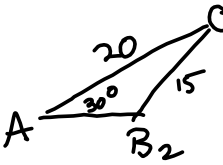
$= \frac{20}{9}\left(\frac{1}{2}\right) = \frac{10}{9} > 1$?? ! No SOL'N!

(c) $\frac{\sin B}{b} = \frac{\sin A}{a} \longrightarrow \frac{\sin B}{20} = \frac{\sin A}{15} = \frac{20\left(\frac{1}{2}\right)}{15} = \frac{10}{15} = \frac{2}{3} = \sin B$

$\boxed{\text{GIVEN: } A = 30^\circ,\ a = 15,\ b = 20}$



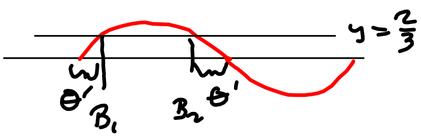
$h = 10$



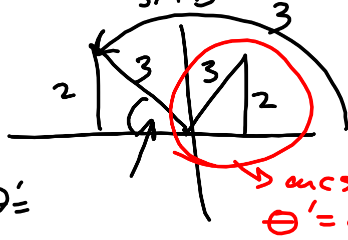
$h < 15 < 20$

$h < a < b \longrightarrow$ 2 sol'ns.

#1e



$\sin B = \frac{2}{3}$



$\theta' =$

$\rightarrow \arcsin\left(\frac{2}{3}\right)$

$\theta' = \arcsin\left(\frac{2}{3}\right)$

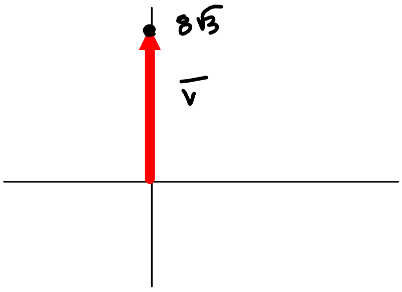
$B_1 = \arcsin\left(\frac{2}{3}\right)$

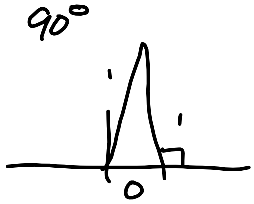
$B_2 = 180^\circ - B_1$

§3.3 #31 NOT ASKED, DINGBAT.

Find the component form of **v** given its magnitude and the angle it makes with the positive x-axis.

Magnitude	Angle
$\|\|\mathbf{v}\|\| = 8\sqrt{3}$	$\theta = 90°$





$$\bar{v} = \|\bar{v}\| \langle \cos\theta, \sin\theta \rangle$$
$$= 8\sqrt{3} \langle \cos 90°, \sin 90° \rangle$$
$$= 8\sqrt{3} \langle 0, 1 \rangle = \langle 0, 8\sqrt{3} \rangle$$

#34 was asked.

Use the Law of Cosines to find the angle $\boldsymbol{\alpha}$ between the vectors. (Assume $0° \le \boldsymbol{\alpha} \le 180°$.)

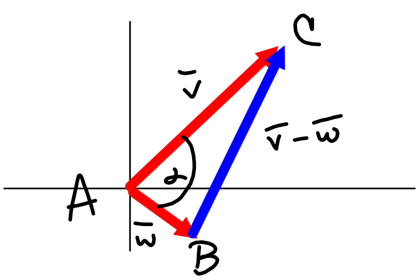
$\mathbf{v} = 7\mathbf{i} + 7\mathbf{j}, \quad \mathbf{w} = \mathbf{i} - \mathbf{j}$

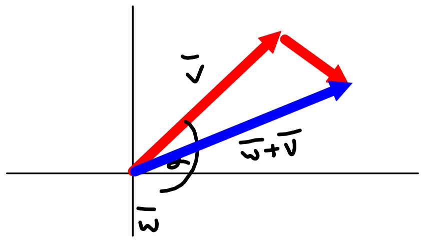
$\bar{v} = \langle 7, 7 \rangle$ $\qquad$ $= \langle 1, 0 \rangle - \langle 0, 1 \rangle$ $\qquad$ $\bar{i} = \langle 1, 0 \rangle$

$\bar{w} = \langle 1, -1 \rangle$ $\qquad$ $\bar{j} = \langle 0, 1 \rangle$

$7\langle 1, 0 \rangle + 7\langle 0, 1 \rangle$

$= \langle 7, 0 \rangle + \langle 0, 7 \rangle = \langle 7, 7 \rangle$





$\bar{v} = \langle 7, 7 \rangle \longrightarrow \|\bar{v}\| = \sqrt{7^2 + 7^2} = \sqrt{49 \cdot 2} = \underbrace{\boxed{7\sqrt{2} = b}}^{= \|\bar{v}\|}$

$\bar{w} = \langle 1, -1 \rangle \longrightarrow \|\bar{w}\| = \sqrt{1^2 + 1^2} = \boxed{\sqrt{2} = c} = \|\bar{w}\|$

$\bar{v} - \bar{w} = \langle 7 - 1, 7 - (-1) \rangle = \langle 6, 8 \rangle \longrightarrow \|\bar{v} - \bar{w}\| = a = \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64} = \sqrt{100} \; \underbrace{\boxed{\pm 10 = a}}_{= \|\bar{v} - \bar{w}\|}$

By the law of cosines

$\cos(\alpha) = \cos(A)$

$a^2 = b^2 + c^2 - 2bc \cos A$

$2bc \cos A = b^2 + c^2 - a^2$

$$\cos A = \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(7\sqrt{2})^2 + (\sqrt{2})^2 - 10^2}{2(7\sqrt{2})(\sqrt{2})}$$

$$= \frac{98 + 2 - 100}{2(7)(2)} = \frac{0}{\sim} = 0$$

$\cos\alpha = 0 \longrightarrow$

$\alpha = \arccos(0) = 90°!$

§3.4 DOT PRODUCT

$\bar{u} = \langle u_1, u_2 \rangle$, $\bar{v} = \langle v_1, v_2 \rangle$ $\longrightarrow$

The dot product of $\bar{u}$ & $\bar{v}$ is

$$\bar{u} \bullet \bar{v} = u_1 u_2 + v_1 v_2$$

(ADVANCED:

$$\bullet : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}$$
$$\langle u_1, u_2 \rangle, \langle v_1, v_2 \rangle$$

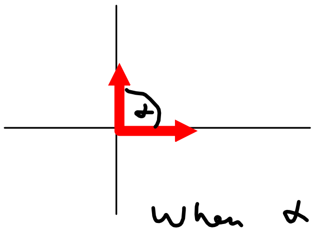
) Don't sweat this.

$$\left(+ : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R} \quad a + b = c \quad 3 + 2 = 5 \right)$$

$$\langle 3,2 \rangle \bullet \langle 5,6 \rangle = 15 + 12 = 27$$

Is closer & closer to zero, the closer the angle is to 90°

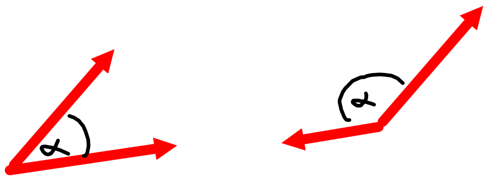
$$\langle 1,0 \rangle \bullet \langle 0,1 \rangle = 1 \cdot 0 + 0 \cdot 1 = 0 + 0 = 0$$



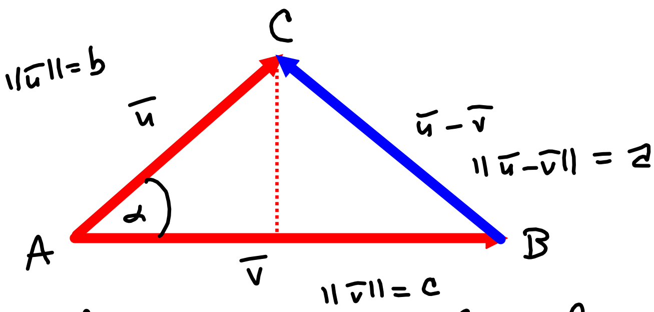
When $\alpha > 90^\circ$, Dot product is negative.

(angle's NEVER greater than 180°

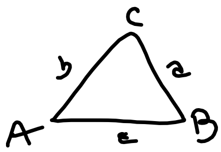
when $0 < \alpha < 90^\circ$, DOT PRODUCT is positive.



α = arccosine, without having to think



We formulate $\cos\alpha$ using law of cosines & dot product



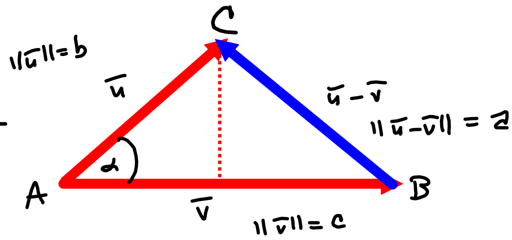
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$\boxed{\text{NOTE}}$ $\|\bar u\| = \sqrt{u_1^2 + u_2^2}$

$$\bar u \cdot \bar u = \langle u_1, u_2\rangle \cdot \langle u_1, u_2\rangle = u_1^2 + u_2^2 = \|\bar u\|^2$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\|\bar u - \bar v\|^2 = \|\bar u\|^2 + \|\bar v\|^2 - 2\|\bar u\|\|\bar v\|\cos A$$



$\to \|\bar u\|\|\bar v\| = \sqrt{\bar u \cdot \bar u}\sqrt{\bar v \cdot \bar v}$

$$(\bar u - \bar v)\cdot(\bar u - \bar v) = \bar u \cdot \bar u + \bar v \cdot \bar v - 2(\bar u \cdot \bar u)(\bar v \cdot \bar v)\cos A$$

$\to \|\bar u\|\|\bar v\|$

$$\bar u \cdot \bar u - \bar u \cdot \bar v - \bar v \cdot \bar u + \bar v \cdot \bar v = \bar u \cdot \bar u + \bar v \cdot \bar v - 2(\bar u \cdot \bar u)(\bar v \cdot \bar v)\cos A$$

$\to \|\bar u\|$

$$(\cos A)\cancel{2}(\bar u \cdot \bar u)(\bar v \cdot \bar v) = \bar u \cdot \bar v + \bar v \cdot \bar u = \cancel{2}\bar u \cdot \bar v \longrightarrow$$

$\to \|\bar v\|$ $\to \|\bar u\|\|\bar v\|$

$$\longrightarrow \cos A = \frac{\bar u \cdot \bar v}{(\bar u \cdot \bar u)(\bar v \cdot \bar v)} = \frac{\bar u \cdot \bar v}{\|\bar u\|^2\|\bar v\|^2}$$

$\to \|\bar u\|\|\bar v\|$

Messed-up, somewhere, Mills!

Should be $\boxed{\dfrac{\bar u \cdot \bar v}{\|\bar u\| \cdot \|\bar v\|} = \cos A}$