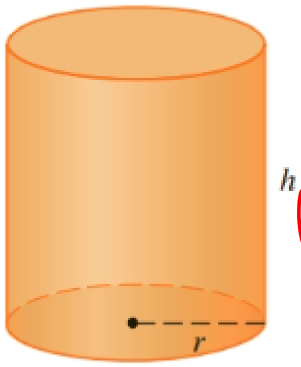
**EXAMPLE 2** A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.




We want to minimize the surface area of this cylinder with top and bottom

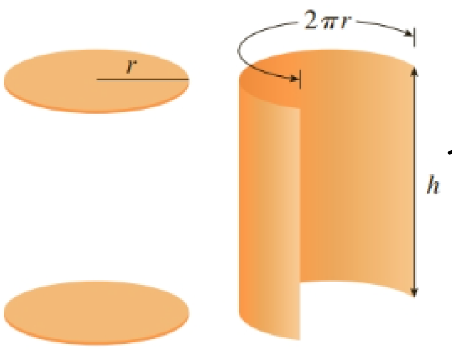
Let $S$ = surface area of the can in $cm^2$,
$r$ = radius of the can (cm), and
$h$ = height of the can (cm)

Lexicon (3 of 10)

**FIGURE 3**

$S$ = area of top & bottom plus area of the wall

$= 2\pi r^2 + 2\pi r h$




Area $2(\pi r^2)$ Area $(2\pi r)h$

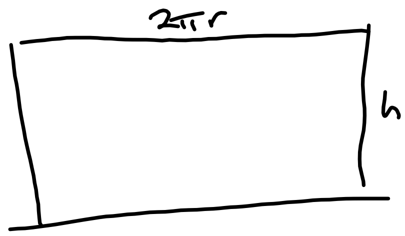



**FIGURE 4**

Fact: Want to hold 1 liter of oil.

$V$ = Volume of can $= \pi r^2 h = 1\,\ell = 1000\,m\ell = 1000\,cm^3$

$\Longrightarrow \pi r^2 h = 1000$ is our AUXILIARY EQUATION

$\Longrightarrow h = \dfrac{1000}{\pi r^2} \Longrightarrow$

$$S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\cancel{\pi} r\left(\frac{1000}{\cancel{\pi} r^2}\right)$$

$$= 2\left[\pi r^2 + \frac{1000}{r}\right] = 2\left[\pi r^2 + 1000 r^{-1}\right] \Longrightarrow$$

$$\frac{dS}{dr} = 2\left[2\pi r - 1000 r^{-2}\right] = 2\left[\frac{2\pi r^3}{r^2} - \frac{1000}{r^2}\right] = 2\left[\frac{2\pi r^3 - 1000}{r^2}\right]$$

$$\underset{=}{\text{SET}}\ 0 \Longrightarrow 2\left[2\pi r^3 - 1000\right] = 0$$

$$\Longrightarrow 2\pi r^3 - 1000 = 0$$

$$\Longrightarrow 2\pi r^3 = 1000$$

$$r^3 = \frac{1000}{2\pi} = \frac{500}{\pi}$$

2 | 500
2 | 250
5 | 125
5 | 25
5

$$\Longrightarrow r = \sqrt[3]{\frac{500}{\pi}} = \boxed{\frac{5\sqrt[3]{4}}{\sqrt[3]{\pi}}}$$ , since $r < 0$ is stupid.

$$= \frac{5\sqrt[3]{4}\sqrt[3]{\pi^2}}{\sqrt[3]{\pi}\cdot\sqrt[3]{\pi^2}} = \frac{5\sqrt[3]{4\pi^2}}{\pi}$$ Rationalized

Another way is to do it implicitly

$S = 2\pi r^2 + 2\pi r h$ $\quad \boxed{\pi r^2 h = 1000}$

$2\pi r^2 + 2\pi r(2r)$

$6\pi r^2 = 1000$

$r^2 = \frac{1000}{6\pi}$

Assume h is implicitly a function of r. $\Rightarrow r = \sqrt{\frac{1000}{6\pi}}$ ?!

Then

$\frac{dS}{dr} = 4\pi r + 2\pi h + 2\pi r h'$

$2\pi r h + \pi r^2 h' = 0$

$2rh + r^2 h' = 0$

$r(2h + rh') = 0$

$2h + rh' = 0$

(Want $\frac{dS}{dr} = 0$)

SET 0 $\longrightarrow$ $2\pi r + \pi h + \pi r h' = 0$

$\Longrightarrow$ $2r + h + rh' = 0$

2 simultaneous eq'ns

$2r + h + rh' = 0$

$2h + rh' = 0$

$\longrightarrow$ $rh' = -2r - h$

$rh' = -2h$

$\Longrightarrow$ $-2r - h = -2h$

$-2r = -h$

$r = \frac{h}{2}$

$h = 2r$

$V = 2\pi r^2 + \pi r^2 h$

$= 2\pi r^2 + \pi r^2(2r)$

$= 2\pi r^2 + 2\pi r^3 = 1000$

$r^2(2\pi + 2\pi r) = 1000$

$2\pi$

Interesting but not a lot of help.