Recall: **Rational Functions from College Algebra**

$$R(x) = \frac{x^2-3x+2}{(x+5)(x-7)} = \frac{(x-2)(x-1)}{(x+5)(x-7)}$$

$x^2-2x-35$

$\mathcal{D} = \mathbb{R} \setminus \{-5, 7\}$

V.A.: $x=-5$, $x=7$

Zeros: (x-ints) $x=1$, $x=2$

x-ints $(1,0)$, $(2,0)$

H.A. (Horizontal Asymptote)

$$\frac{x^2-3x+2}{x^2-2x-35} \xrightarrow{x \to \infty} \frac{x^2}{x^2} = 1 = y$$

$$\frac{x^2\left(1-\frac{3}{x}+\frac{2}{x^2}\right)}{x^2\left(1-\frac{2}{x}-\frac{35}{x^2}\right)} \xrightarrow{x \to \infty} \frac{1-0+0}{1-0-0} = 1 = y$$

($\frac{3}{x} \to 0$, $\frac{2}{x^2} \to 0$, $\frac{2}{x} \to 0$, $\frac{35}{x^2} \to 0$)

PROPER RATIONAL Functions:

The degree of the denominator is greater than the degree of the numerator.

$\lim_{x \to \pm\infty} (\text{PROPER}) = 0$

$\frac{x^2 + 22x}{x^5 - 7} \xrightarrow{x \to \infty} \boxed{0 = y}$

SLANT/OBLIQUE Asymptotes.

$\frac{x^3 + 5x^2 - 7x + 1}{x + 1}$

Divide: → OBLIQUE ASYMPTOTE

$x^2 + 4x - 11$

$x+1 \;\big|\; x^3 + 5x^2 - 7x + 1$

$-(x^3 + x^2)$

$4x^2 - 7x + 1$

$-(4x^2 + 4x)$

$-11x + 1$

$\frac{x^3}{x} = x^2$

$\frac{4x^2}{x} = 4x$

$-1 \;\big|\; 1 \quad 5 \quad -7 \quad 1$

$\quad\quad -1 \quad -4 \quad 11$

$\quad 1 \quad 4 \quad -11 \quad | \; 12$ → $r = 12$

$x^2 + 4x - 11$ $\quad r\ 12$

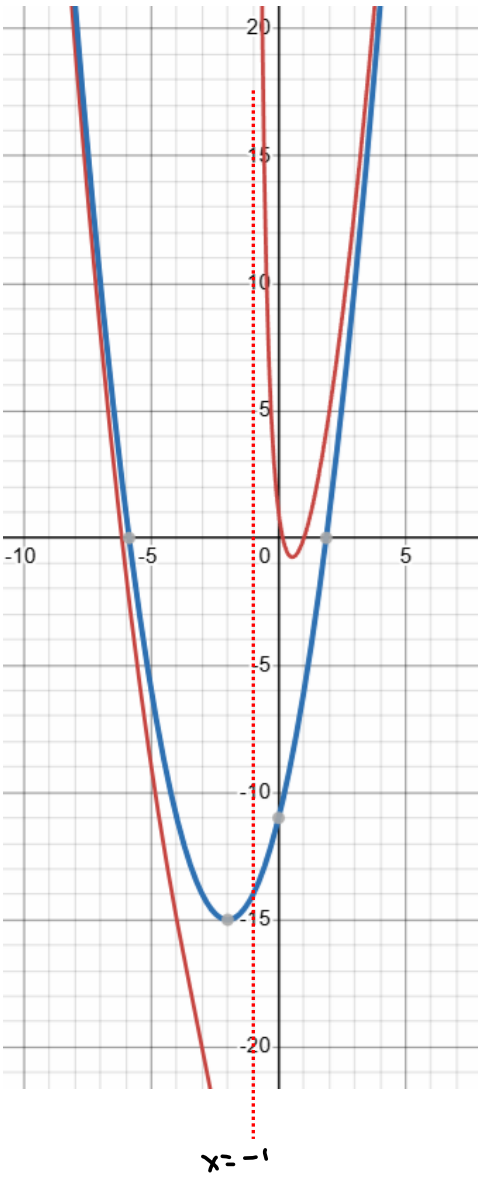
$$f(x) = \frac{\left(x^3 + 5x^2 - 7x + 1\right)}{x + 1}$$

$$x^2 + 4x - 11$$

Desmos graph of *f* and its oblique asymptote.

These get messy. Standard one is the easiest type. quadratic divided by linear polynomial.

$$\frac{x^2+5x+6}{x-2} = \frac{(x+2)(x-3)}{x-2}$$



$\frac{x^2+5x+6}{x-2} = \frac{(x+2)(x-3)}{x-2} = \frac{x^2-x-6}{x-2}$ NOPE!

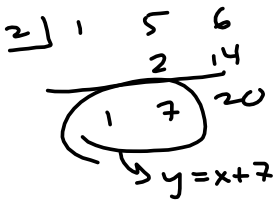
$D = \mathbb{R} \setminus \{2\}$

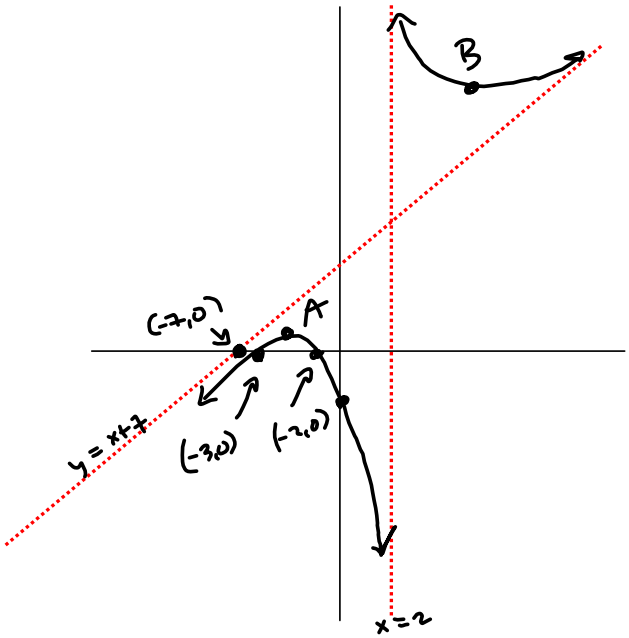
V.A.: $x=2$

x-ints: $(-2,0), (3,0)$

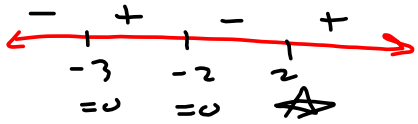
H.A.: None

O.A.:





$\frac{(x+2)(x+3)}{x-2}$



Use Calculus to find the local extrema.

$\frac{f'g - fg'}{g^2}$

$R(x) = \frac{x^2+5x+6}{x-2} \Rightarrow R'(x) = \frac{(2x+5)(x-2) - (x^2+5x+6)(1)}{(x-2)^2}$

$= \frac{2x^2+x-10-x^2-5x-6}{(\ )^2} = \frac{x^2-4x-16}{(x-2)^2} = R'(x)$

SET $= 0 \Rightarrow x^2 - 4x - 16 = 0 \Rightarrow$

$x^2 - 4x + 2^2 - 4 - 16$

$= (x-2)^2 - 20 = 0$

$\Rightarrow (x-2)^2 = 20$

$x - 2 = \pm\sqrt{20} = \pm 2\sqrt{5}$

$\Rightarrow x = 2 \pm 2\sqrt{5}$

$A = (2 - 2\sqrt{5}, R(2-2\sqrt{5}) \approx (-2.47, .06)$

$B = (2 + 2\sqrt{5}, R(2+2\sqrt{5}) \approx (6.47, 17.94)$

$f(2 - 2\sqrt{5}) = 0.0557280900008$

$f(2 + 2\sqrt{5}) = 17.94427191$

$\frac{x^2 - 4x - 16}{(x-2)^2}$ $= R'(x)$

$\frac{40}{(x-2)^3} = R''(x)$