

I didn't show you how to use Test Values. Also, I always lay out the intervals as intervals, rather than compound inequalities.

Recall, we were working on

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$

$$\Rightarrow f'(x) = 12x^3 - 12x^2 - 24x \stackrel{SET}{=} 0$$

$$\Rightarrow 12x(x^2 - x - 2) = 0$$

$$\Rightarrow 12x(x-2)(x+1) = 0$$

$$\Rightarrow x = -1, 0, 2 \quad (\text{CRITICAL \#s})$$

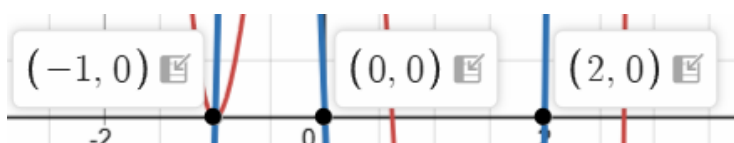
$12x^3$  ...  $\uparrow$   
 $\downarrow$  End Behavior

Use  $g(x)$  for  $f'(x)$  on Desmos.



| Interval        | Test           |
|-----------------|----------------|
| $(-\infty, -1)$ | -2             |
| $(-1, 0)$       | $-\frac{1}{2}$ |
| $(0, 2)$        | 1              |
| $(2, \infty)$   | 3              |

Snapshot of the zeros of  $f'$ .

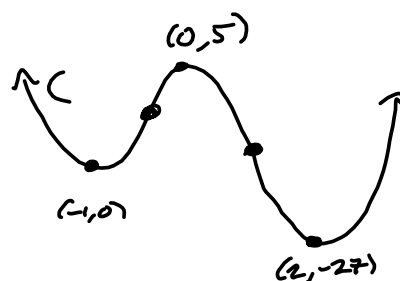


|                                  |          |
|----------------------------------|----------|
| $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ | $\times$ |
| $g(x) = 12x^3 - 12x^2 - 24x$     | $\times$ |
| $f(0)$                           | $\times$ |
|                                  | = 5      |
| $f(-1)$                          | $\times$ |
|                                  | = 0      |
| $f(2)$                           | $\times$ |
|                                  | = -27    |

$(0, 5)$  MAX

$(-1, 0)$  MIN

$(2, -27)$  MIN



From 1<sup>st</sup> Derivative.

Now let's see the influence of the 2<sup>nd</sup> derivative.

Snapshot of the zeros of  $f$ .



$$f'(x) = 12x^3 - 12x^2 - 24x \rightarrow$$

$$f''(x) = 36x^2 - 24x - 24 \stackrel{!}{=} 0$$

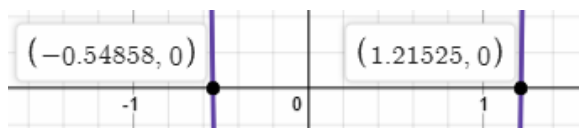
$$\rightarrow 12(3x^2 - 2x - 2) = 12(3x \quad)(x \quad) \text{ DOH!}$$

$$b^2 - 4ac = 2^2 - 4(3)(-2) = 4 + 24 = 28$$

$$\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{28}}{2(3)} = \frac{2 \pm 2\sqrt{7}}{6}$$

$$= \frac{1 \pm \sqrt{7}}{3} \rightarrow \begin{cases} \frac{1 + \sqrt{7}}{3} \approx 1.21525 \\ \frac{1 - \sqrt{7}}{3} \approx -0.54858 \end{cases}$$

Snapshot of the zeros of  $f''$ .



$$f''(x) = 36(x - (\frac{1 + \sqrt{7}}{3}))(x - (\frac{1 - \sqrt{7}}{3}))$$

Confirming our hand work.

$$h(x) = 36x^2 - 24x - 24$$

$$\frac{(1 + \sqrt{7})}{3}$$

$$= 1.21525043702$$

$$\frac{(1 - \sqrt{7})}{3}$$

$$= -0.548583770355$$

$$f(1.21525043702)$$

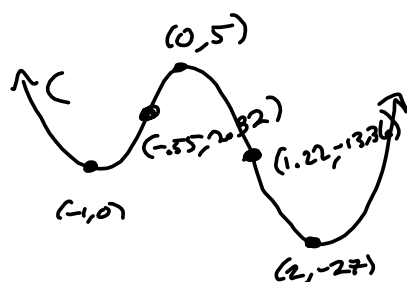
$$= -13.3577816624$$

$$(1.2152504, -13.3577816624)$$

$$f(-0.548583770355)$$

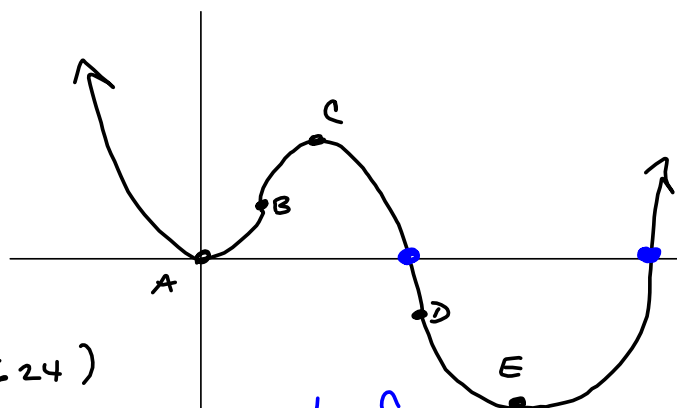
$$= 2.32074462538$$

$$(-0.548583770355, 2.32074462538)$$



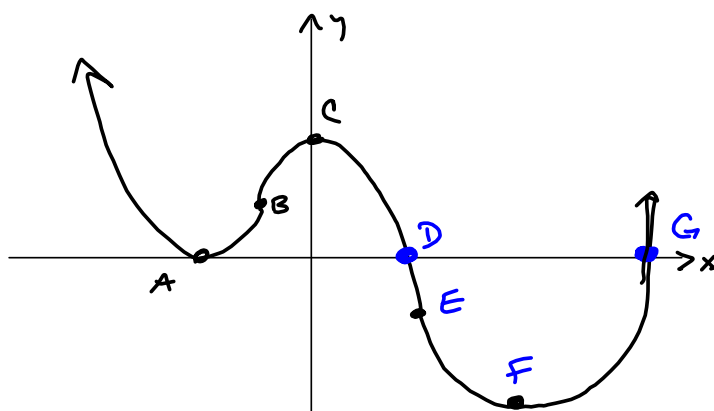
$(1.2152504, -13.3577 \approx 16624)$

$(-0.5486, 2.3207446)$



x-ints of  
 $f(x)$  and the  
 part calculus doesn't  
 help us with

"COMPLETE GRAPH"  
 includes everything:  
 x-ints  
 MAX/MIN  
 Asymptotes  
 Inflection Points (concavity)  
 ☺ ☺



$$\begin{aligned}
 A &= (-1, 0) \text{ MIN} \\
 B &= (-.55, 2.32) \text{ IP} \\
 C &= (0, 5) \text{ MAX} \\
 D &= (.61, 0) \text{ X-INT (TECH)} \\
 E &= (1.22, -13.36) \text{ IP} \\
 F &= (2, -27) \text{ MIN} \\
 G &= (2.72, 0) \text{ X-INT}
 \end{aligned}$$

I didn't really analyze the  $f''$  for sign changes. I just knew it was a polynomial, and this is what they look like.

here's the  $f''$  analysis: Test

$$(-\infty, \frac{1-\sqrt{2}}{3}) \approx (-\infty, -.55)$$

$$(\frac{1-\sqrt{2}}{3}, \frac{1+\sqrt{2}}{3}) \approx (-.55, 1.22)$$

$$(\frac{1+\sqrt{2}}{3}, \infty) \approx (1.22, \infty)$$

$f''$  is quadratic Polynomial

$$\begin{array}{c}
 f'' \\
 \begin{array}{c}
 + \quad \quad - \quad \quad + \\
 \smile \quad \frac{1-\sqrt{2}}{3} \quad \smile \quad \frac{1+\sqrt{2}}{3} \quad \smile
 \end{array}
 \end{array}$$